

SEENET-MTP WORKSHOP

# BW2021

Beyond the Standard Model(s)  
Field Theories and the Early Universe



## THE BOOK OF EXTENDED ABSTRACTS

Editors:

Marija Dimitrijević Ćirić, Dragoljub D. Dimitrijević,  
Goran S. Djordjević and Milan Milošević

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# **BW2021**

**SEENET-MTP Workshop**

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## PREFACE

The SEENET-MTP Workshop Beyond the Standard Model(s) - Balkan Workshop (BW2021), <https://bw2021.seenet-mtp.info>, was held in Belgrade, Serbia, 7 - 10 September, 2021. The BW2021 was the main event of the SEENET-MTP network (<http://www.seenet-mtp.info/>) program in 2021. Despite Covid-19 pandemic the BW2021 Workshop took place fully on site, following the existing health measures – at the Faculty of Physics, University of Belgrade, Serbia.

The main organizers were SEENET-MTP Network – Centre, Faculty of Science and Mathematics, University of Niš (Serbia) and Faculty of Physics, University of Belgrade in cooperation with the International Centre for Theoretical Physics ICTP (Trieste, Italy).

The success of the previous Balkan Workshops (BW2003, BW2005, BW2007, SSSCP2009, BSW2011, BW2013 and BSW2018), and other numerous SEENET-MTP schools and conferences, has been a good motivation to proceed with this event, but in a more complex and delicate circumstances.

The BW2021 Workshop, entitled “Beyond the Standard Model(s)” covers some open problems in particle physics and cosmology, but also mathematical physics, the newer topics as quantum information, and, as always, pointed on (inter)regional cooperation. In total more than 30 participants and guests from 8 countries, and 16 lecturers continue tradition of high level scientific events in the region through the SEENET-MTP framework.

The present Book of Extended Abstracts contain all contributions submitted to the BW2021 event. The contributions for the Workshop are listed below in alphabetical order of the first author. The editors have done minimal, and technical corrections.

Talks presented at the Workshop were recorded and are available at official webpage.

We would like to thank Dr. Nikola Konjik and Dr. Marko Stojanović for their great help in preparing publications and the whole event.

We kindly acknowledge the financial and other support from the following institutions:

- ICTP (The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy)
- EPS (European Physical Society, Mulhouse, France)
- CERN (Geneva, Switzerland)
- Faculty of Sciences and Mathematics, University of Nis (Serbia), as well as
- Serbian Ministry of Education, Science and Technological Development, Belgrade which contribution is announced.

We would like to mention excellent collaboration with the Faculty of Physics, University of Belgrade, Faculty of Sciences and Mathematics, University of Niš, and the Serbian

Academy of Sciences and Arts in preparation and organization of the BW2021, as well as for the satellite BPU-EPS Meeting <https://bpu-eps2021.seenet-mtp.info/>.

The organizers would like to thank all the participants of the BW2021 who came from abroad despite the Covid-19 pandemic.

Belgrade, Niš  
October, 2021

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# ENTROPY OF REISSNER-NORDSTRÖM-LIKE BLACK HOLES

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## 1 Introduction

The concept of entropy represents an essential ingredient of the thermodynamic behavior of black holes. It can be classically represented by a boundary term  $\Gamma_H$ , interpreted as the Noether charge on black hole horizon [1]. For a class of spherically symmetric black holes in GR without matter,  $\delta\Gamma_H$  has the standard form  $T\delta S$ , where  $T$  is the black hole temperature and  $S$  entropy, equal to one-fourth of the horizon area. Such a universal nature of  $S$  is kind of a puzzle, as one might naturally expect that entropy would depend on the structure of black holes and the type of the underlying gravitational dynamics [2].

Entropy and the asymptotic charges (such as energy or angular momentum) are strongly interrelated through the first law of black hole thermodynamics. Recently, a Hamiltonian approach to entropy was proposed in [3], in which the asymptotic charges and entropy are described in a unified manner as the canonical charges at infinity and horizon, respectively. This approach was primarily intended to describe entropy in the framework of Poincaré gauge theory (PG), where both the curvature and the torsion are essential ingredients of the gravitational dynamics. However, for a number of black holes in PG, including Kerr-AdS black holes *with or without torsion* [4], it was found, somewhat unexpectedly, that entropy retains its area form, and the first law remains unchanged.

Are there black holes for which entropy and/or the first law change their standard forms? There are two 3-dimensional PG models which are interesting in this respect. First, entropy of the Banados-Teitelboim-Zanelli black hole with torsion is found to depend on the parameter that measures the strength of torsion, and second, entropy of the Oliva-Tempo-Troncoso black hole depends on a “hair” parameter appearing in the metric. In both cases, the first law remains unchanged. This situation motivates us to explore thermodynamic aspects of a new black hole with torsion, found in the four-dimensional PG by Cembranos and Valcarcel [5].

## 2 Reissner-Nordström-like black holes

The solution [5] represents a *gravitational analogue* of the standard Reissner-Nordström black hole, in which the “electric charge” is produced not by a source, as in Einstein-Maxwell theory, but by the *gravitational field in vacuum*. The static metric of asymptotically flat Reissner-Nordström-like black holes is characterized by the shift function

$$N^2 = 1 - \frac{2m}{r} + \frac{q}{r^2},$$

parametrized by  $m$ , the usual mass parameter of the Schwarzschild metric, and  $q$ . Parameter  $q$  can be expressed (on shell) in terms of the Lagrangian coupling constants and a particular torsion parameter  $p$ . The horizon is located at  $r = r_+$ , the larger root of the equation  $N^2 = 0$ .

## 3 Black hole thermodynamics

By using the Hamiltonian approach [3] one obtains the energy  $E$  of the solution, by varying the canonical generator at spatial infinity  $\delta E = \delta \Gamma_\infty = \delta m$ . Entropy of the black hole is determined by the variation of the boundary term at horizon. However,  $S$  cannot be obtained from the usual formula  $T \delta S = \delta \Gamma_H$ , as it does not define  $S$  as a local function of the thermodynamic variables  $(m, q)$ . The problem is resolved by focussing on the  $\delta$ -integrable part of  $\delta \Gamma_H$ , which implies that entropy is one-fourth of the horizon area  $S = \pi r_+^2$ .

At the same time, the standard form of the first law is modified by an extra contribution stemming from the hair parameter  $q$ :

$$\delta m = T \delta S + \frac{1}{2r_+} \delta q.$$

We plan to extend our present analysis of Reissner-Nordström-like black holes to the most general Poincaré gauge theory, with all possible parity even and parity odd modes.

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## JACOBI-LIKE STRUCTURES IN THE LINE BUNDLE LANGUAGE

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### 1 Introduction

Poisson-like concept, one of the strongest pillars of modern physics at classical and quantum levels, has recently known an increasing interest generated by string field theory [1] and integrability [2]. From the integrability perspective, Poisson-like structures display completely integrable (Stefan-Sussmann) characteristic distributions with characteristic leaves symplectic-like manifolds.

In the middle of the '70s, the Poisson structure concept was 'improved' to what is now known as a Jacobi structure [3]. This encompasses (via the completely integrability of its characteristic distribution) both locally conformal symplectic manifolds and contact manifolds. Initially, a Jacobi structure on a given manifold  $M$  has been given in terms of a bivector  $\Pi \in \mathfrak{X}^2(M)$  and a vector field  $E \in \mathfrak{X}^1(M)$  (defining a Jacobi pair  $(\Pi, E)$ ) subject to the consistency equations

$$\frac{1}{2}[\Pi, \Pi] + E \wedge \Pi = 0, \quad [E, \Pi] = 0,$$

where  $[\cdot, \cdot]$  is the Schouten-Nijenhuis bracket in the Gerstenhaber algebra  $(\mathfrak{X}^\bullet(M), \wedge, [\cdot, \cdot])$  of multivector fields. This structure is equivalent to a local  $\mathbb{R}$ -Lie algebra structure on the set of real smooth functions  $\mathcal{F}(M)$

$$\{f, g\}_{(\Pi, E)} := \Pi(f, g) + fE(g) - gE(f).$$

Using the identification

$$\mathcal{F}(M) = \Gamma(\mathbb{R}_M), \quad \mathbb{R}_M := \mathbb{R} \times M,$$

Jacobi concept is straightforwardly implemented for a generically (non-trivial) line bundle  $L \rightarrow M$  in terms of a local  $\mathbb{R}$ -Lie algebra structure on the set of smooth sections in the line bundle  $\Gamma(L)$ ,  $\{\cdot, \cdot\}$ , or, equivalently, in terms of a bidifferential operator  $J \in \mathcal{D}^2L$  subject to Maurer-Cartan equation

$$[[J, J]] = 0,$$

where  $[[\cdot, \cdot]]$  is the Gerstenhaber-Jacobi bracket in the module  $\mathcal{D}^\bullet L$ .

The present talk focuses on Jacobi bundles with background. By its very definition, a Jacobi bundle with background [4] is a line bundle  $L \rightarrow M$  equipped with Jacobi structure

with background which consists of a first-order bidifferential operator  $J$  and an  $L$ -valued Atiyah 3-form  $\Phi \in \Omega_L^3$  subject to the consistency equation

$$\frac{1}{2}[[J, J]] = \wedge^3 \hat{J}\Phi.$$

Previously, by  $\hat{J}$ , we meant the linear ‘prolongation’ of the bidifferential operator  $J$ ,  $\hat{J} \in \Gamma(\wedge^2 J_1 L \otimes L)$ . When the line bundle is trivial,  $L = \mathbb{R}_M$ , the Jacobi structure with background reduces [5] to a Jacobi pair with background  $((\Pi, E), (\phi, \omega))$  verifying the consistency conditions

$$\frac{1}{2}[\Pi, \Pi] + E \wedge \Pi = \Pi^\sharp \phi + \Pi^\sharp \omega \wedge E, \quad [E, \Pi] = -\left(\Pi^\sharp i_E \phi + \Pi^\sharp i_E \omega \wedge E\right).$$

## 2 Conclusion

The analysis aiming Jacobi pairs with background reveals the following:

- they display completely integrable characteristic distributions with characteristic leaves either locally conformal symplectic manifolds with background or twisted contact manifolds;
- they are in one-to-one correspondence with ‘homogeneous’ Poisson structures with (non-necessarily closed) three form.

In the general line bundle setting, it is shown that transitive Jacobi structures with background are either twisted contact structures (for odd-dimensional base manifolds) or locally conformal symplectic with background structures (for even-dimensional base manifolds).

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**ABOUT TACHYON INFLATION IN  
THE HOLOGRAPHIC BRANEWORLD**

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The cosmological inflation scenario successfully solves the horizon and other related problems of the standard Big Bang cosmology. Tachyon models belong to a class of inflationary models where inflation is driven by the tachyon scalar field originating in M or string theory [1]. The effective field theory Lagrangian is of the DBI form [2]

$$\mathcal{L} = -\ell^{-4}V(\theta/\ell)\sqrt{1 - g^{\mu\nu}\theta_{,\mu}\theta_{,\nu}}, \quad (1)$$

where  $\ell$  is an appropriate length scale,  $V$  is dimensionless tachyon potential,  $\theta_{,\mu} = \partial\theta/\partial x^\mu$  and the tachyon field  $\theta$  is of dimension of length. We study a braneworld inflation model in the framework of a holographic cosmology [3]. This cosmological model is based on the effective four-dimensional Einstein equations on the holographic boundary in the framework of anti-de Sitter/conformal field theory (AdS/CFT) correspondence. A D3-brane is located at the holographic boundary of an asymptotic ADS5 bulk [4].

An interesting property of the model is that the universe evolution starts from a point at which the energy density and cosmological scale are both finite, bypassing the Big Bang singularity of the standard cosmology. Assuming that the holographic braneworld is spatially flat FRW universe and employing the holographic Friedmann equations

$$h^2 - \frac{1}{4}h^4 = \frac{\kappa^2}{3}\ell^4\rho, \quad (2)$$

$$\dot{h}\left(1 - \frac{1}{2}h^2\right) = -\frac{\kappa^2}{2}\ell^3(p + \rho), \quad (3)$$

where  $\rho$  and  $p$  are density and pressure of a cosmological fluid, respectively,  $h \equiv \ell H$  is dimensionless expansion rate, while  $\kappa^2 = 8\pi G_N \ell^{-2}$  is the fundamental dimensionless coupling, we

finally end up with the solution of the form

$$h^2 = 2 \left( 1 - \sqrt{1 - \frac{\kappa^2}{3} \ell^4 \rho} \right). \quad (4)$$

Hubble expansion rate is between zero and the maximal value  $h_{\max} = \sqrt{2}$ , corresponding to the maximal energy density  $\rho_{\max} = 3/(\kappa^2 \ell^4)$ . In this way, the inflation phase proceeds immediately after the initial moment [4], as it is usually the case in the modified cosmological models.

In addition, for inflation to be predictive, one needs to ensure that inflation is independent of initial conditions. That is, one should ensure that there is an attractor solution to the dynamics, such that differences between solutions corresponding to different initial conditions rapidly vanish [5].

Starting from the classical background and spatially homogenous tachyon field  $\theta(t)$ , the equation of motion in the phase plane  $(\theta, \dot{\theta})$ , i.e. defining equation for the phase space trajectory of the tachyon field has general form

$$\frac{d\dot{\theta}}{d\theta} = -\frac{1}{\dot{\theta} p_{,\dot{\theta}\dot{\theta}}} \left[ 3 \frac{h}{\ell} p_{,\dot{\theta}} + \dot{\theta} p_{,\dot{\theta}\theta} - p_{,\theta} \right], \quad (5)$$

For the tachyon case, in the slow-roll regime, equation for the attractor trajectory is derived from the previous one, leading to the expression for deviation of the expansion rate  $h$  in the form

$$\delta h(\theta) = \delta h(\theta_i) e^{-3N}, \quad (6)$$

where  $N$  is the number of e-folds. We may conclude that regardless of the initial condition, the attractor behaviour implies that late-time solutions are the same up to a constant time shift, which cannot be measured [5].

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# L-INFINITY ALGEBRA AND BRAIDED FIELD THEORY

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## 1 Introduction

$L_\infty$ -algebra (strong homotopy algebra) is a generalization of a Lie algebra with higher order brackets. Recently it was realized that any classical field theory with generalized gauge symmetries is determined by an  $L_\infty$ -algebra, due to duality with BV-BRST [1]. In our previous work [2] we deformed the classical  $L_\infty$ -algebra to a braided  $L_\infty$ -algebra using the Drinfeld twist formalism. From the braided  $L_\infty$ -algebra, the braided field theory can be constructed. It is a noncommutative deformation of the usual (gauge) field theory, but compared to the previously discussed NC field theories it has some new features.

After a general discussion, in this talk we present one particular example of the braided field theory:  $U(1)$  gauge theory coupled to a Dirac fermion, braided electrodynamics. In particular, we will comment on differences between our model and previously constructed models of NC electrodynamics.

## 2 Braided Electrodynamics

Starting from a suitable classical  $L_\infty$ -algebra and using the Drinfeld twist deformation quantization [3] one can obtain a braided  $L_\infty$ -algebra. Given any Drinfeld twist  $\mathcal{F}$ , we deform the brackets  $\ell_n$  to twisted brackets  $\ell_n^*$  such that they braided symmetric

$$\ell_n^*(\dots, v, v', \dots) = -(-1)^{|v||v'|} \ell_n^*(\dots, R_k(v'), R^k(v), \dots). \quad (1)$$

The braided brackets  $\ell_n^*$  fulfil braided homotopy relations. For example, the third relation  $\mathcal{S}_3(v_1, v_2, v_3) = 0$  is given by

$$\begin{aligned} & \ell_2^*(\ell_2^*(v_1, v_2), v_3) - (-1)^{|v_2||v_3|} \ell_2^*(\ell_2^*(v_1, R_k(v_3)), R^k(v_2)) \\ & + (-1)^{(|v_2|+|v_3|)|v_1|} \ell_2^*(\ell_2^*(R_k(v_2), R_l(v_3)), R^l R^k(v_1)) - \ell_1^*(\ell_3^*(v_1, v_2, v_3)) \\ & = -\ell_3^*(\ell_1^*(v_1), v_2, v_3) - (-1)^{|v_1|} \ell_3^*(v_1, \ell_1^*(v_2), v_3) - (-1)^{|v_1|+|v_2|} \ell_3^*(v_1, v_2, \ell_1^*(v_3)). \end{aligned} \quad (2)$$

We see that the non-trivial braiding, expressed in terms of R-matrices, is included in this relation.

A braided field theory can be fully defined in terms of its braided  $L_\infty$ -algebra. Let  $(V, \{\ell_n\})$  be a 4-term braided  $L_\infty$ -algebra. For a gauge parameter  $\rho \in V_0$ , we define the braided gauge variation of a field  $A \in V_1$  by

$$\delta_\rho A = \ell_1^*(\rho) + \sum_{n=1}^{\infty} \frac{1}{n!} (-1)^{\frac{1}{2}n(n-1)} \ell_{n+1}^*(\rho, A, \dots, A). \quad (3)$$

Braided covariant dynamics is described by the equations of motion

$$F_A = \sum_{n=1}^{\infty} \frac{1}{n!} (-1)^{\frac{1}{2}n(n-1)} \ell_n^*(A, \dots, A). \quad (4)$$

Finally, using the (strictly) cyclic pairing we can define an action for our braided field theory as

$$S(A) = \sum_{n=1}^{\infty} \frac{1}{(n+1)!} (-1)^{\frac{1}{2}n(n-1)} \langle A, \ell_n^*(A, \dots, A) \rangle, \quad (5)$$

As a simple example we discuss the  $U(1)$  gauge field  $A_\mu(x)$  coupled to the massless Dirac fermion  $\psi$  on the 4D Minkowski space-time. The corresponding braided  $L_\infty$ -algebra is given by

$$\begin{aligned} \ell_1^*(\rho) &= \begin{pmatrix} 0 \\ 0 \\ \partial_\mu \rho \end{pmatrix}, \quad \ell_1^*(\mathcal{A}) = \begin{pmatrix} i\gamma^\mu \partial_\mu \psi \\ -i\gamma^\mu \partial_\mu \bar{\psi} \\ \partial_\mu \partial_\nu A^\nu - \partial_\nu \partial^\nu A_\mu \end{pmatrix}, \quad \ell_1^*(F_{\mathcal{A}}^*) = \partial_\mu F_A^*. \\ \ell_2^*(\rho, \mathcal{A}) &= \begin{pmatrix} -iR_k(\bar{\psi}) \star R^k(\rho) \\ i\rho \star \psi \\ i[\rho, A]_\star = 0 \end{pmatrix}, \quad \ell_2^*(\mathcal{A}_1, \mathcal{A}_2) = -\frac{1}{2} \begin{pmatrix} \gamma^\mu A_{1\mu} \star \psi_2 + R_k \gamma^\mu A_{2\mu} \star R^k \psi_1 \\ \bar{\psi}_1 \star \gamma^\mu A_{2\mu} + R_k \bar{\psi}_2 \star \gamma^\mu R^k A_{1\mu} \\ 0 \end{pmatrix}. \end{aligned} \quad (6)$$

Here we defined a "master" field  $\mathcal{A} \in V_1$  and the corresponding equations of motion  $F_{\mathcal{A}} \in V_2$  as

$$\mathcal{A} = \begin{pmatrix} \bar{\psi} \\ \psi \\ A \end{pmatrix}, \quad F_{\mathcal{A}} = \begin{pmatrix} F_{\bar{\psi}} \\ F_\psi \\ F_A \end{pmatrix}. \quad (7)$$

This algebra results in the action

$$S = \int d^4x \left\{ -\frac{1}{4} F^{\mu\nu} \star F_{\mu\nu} + \bar{\psi} \star i\gamma^\mu \partial_\mu \psi + \frac{1}{2} (\bar{\psi} \star A_\mu \gamma^\mu \star \psi + \bar{\psi} \star R_k(A_\mu) \gamma^\mu \star R^k(\psi)) \right\}. \quad (8)$$

From the braided II Noether identity the braided conserved charge follows

$$Q^* = \int d^3\vec{x} (\psi^\dagger \star \psi + R_k(\psi^\dagger) \star R^k(\psi)). \quad (9)$$

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# THERMODYNAMICS AND COMPLEXITY OF HOLOGRAPHIC BACKGROUNDS

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## 1 Introduction

Recently there has been a progress in thermodynamics of black holes, where one can consider the cosmological constant  $\Lambda$  as a thermodynamic parameter [1]. In this extended thermodynamics the conjugate variable to  $\Lambda$  is  $\Theta$ . The first law of black hole thermodynamics now yields:

$$dM = TdS + \Omega_i dJ_i + \Phi_i dQ_i + Vdp \quad (1)$$

where  $p = -\Lambda/(8\pi)$  and  $V = -8\pi\Theta$  are the thermodynamic pressure and the thermodynamic volume of the black hole.

In general, one can calculate  $p$  and  $V$  through the Killing two form potential  $\omega$ , such that  $\xi = \star d\star\omega$ , where  $\xi = \xi_{(t)} + \Omega\xi_{(\varphi)}$  is the standard stationary Killing vector of the background. The equation for  $\omega$  is not enough to completely fix it, thus a gauge condition is required. In this case, one can use the Komar integral for the mass  $M$  of the black hole,

$$M = -\frac{D-2}{16\pi(D-3)} \int (\star d\xi_{(t)} + 2\Lambda\star\omega), \quad (2)$$

in order to fix the gauge for  $\omega$ . The latter works only if  $M$  was derived by other means. After fixing the gauge for the Killing two form potential  $\omega$ , by definition one has:

$$d\star d\xi + 2\Lambda d\star\omega, \quad \Theta = \frac{D-2}{16\pi} \int \star\omega. \quad (3)$$

The integration can be done over the horizon of the black hole.

Considering a given black hole solution one can go further and calculate the complexity growth rate  $\dot{C}$  of the system [2, 3]. At late times this quantity saturates the so called Lloyd bound [4]:

$$\dot{C} = (F + TS)_+ - (F + TS)_-, \quad (4)$$

where the gravitational action growth can be written as the difference of the generalized enthalpy between the two corresponding horizons. Here  $F$  is the free energy. An interesting situation occurs when the inner Cauchy horizon is replaced by a physical curvature singularity. In this case (4) can be used to estimate how far one can probe the interior of the black hole.

In the present talk we study the extended thermodynamics and the complexity growth rate of 3d holographic models with a cosmological parameter. Our investigation is conducted within the "complexity equals action" conjecture for black holes with horizons and in the presence of physical curvature singularities. We also show how far from the curvature singularity one still has a well defined theory.

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## 4D NC GRAVITY FROM NC 5D CS THEORY

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### 1 Introduction

Chern-Simons (CS) Lagrangians play an important role in many different areas of contemporary physics. In  $D = 2n + 1$  dimensional space-time, this Lagrangian is constructed from a (quasi) gauge invariant top form that does not depend on a space-time metric. In order to construct it, one should know a gauge group, symmetric invariant form  $\langle \dots \rangle_{n+1}$  on its Lie algebra, and from the identity

$$\langle F, \dots, F \rangle_{n+1} = dQ, \quad (1)$$

where  $F$  is a curvature for a given connection  $A$ , obtain Chern-Simons Lagrangian as  $L = kQ$ , for some constant  $k$ . In the case where the gauge group is chosen to be dS (AdS)  $SO(5, 1)$  ( $SO(4, 2)$ ), and in any odd number of dimensions, it can be shown that for the connection  $A = \frac{1}{2}\omega^{AB}J_{AB} + \frac{1}{7}e^AJ_{A5}$  (where spin connection and vielbein are combined into a single (A)dS connection), one obtains a Lagrangian that corresponds to a special case of Lovelock Gravity Theories<sup>1</sup> [1]. Having said this, we can compute that 5D CS Lagrangian in the case of  $SO(4, 2)$  can be written as [2]:

$$L_{CS}^5 = const \cdot \varepsilon_{ABCDE} \left( \frac{1}{5!5} e^A e^B e^C e^D e^E + \frac{2}{3!3} R^{AB} e^C e^D e^E + \frac{1}{7} R^{AB} R^{CD} e^E \right). \quad (2)$$

On the other hand, Noncommutative (NC) Field Theory is believed by some to represent a correct effective theory that captures (some) quantum properties of gravity. Moreover, to construct gravity theories on a noncommutative space-time, one usually starts with a commutative gauge theory defined solely in terms of its connection, which can be related to gravity by identifying the connection components with gravity variables in the first-order formalism. Thus it is interesting to find an NC version of the 5D CS theory. This was done by Aschieri and Castellani in [3] using geometric Seiberg-Witten map. Noncommutativity is introduced by abelian Drinfeld twist. This means that the algebra of functions on ordinary space-time is deformed via NC star product  $f \star g = \mu\{\mathcal{F}^{-1}f \otimes g\}$ , where we have  $\mu\{f \otimes g\} = f \cdot g$  and

$$\mathcal{F}^{-1} = e^{\frac{i}{2}\theta^{IJ}X_I \otimes X_J}, \quad (3)$$

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<sup>1</sup>Invariant symmetric form on a Lie algebra is defined using the Trace in a certain representation

for commuting vector fields  $X_I$ , and constant antisymmetric matrix  $\theta^{IJ}$ . More generally, one can deform exterior product between arbitrary (Lie algebra-valued) forms and obtain an NC action invariant under deformed  $SO(4,2)$  gauge transformations that can be perturbatively expanded in powers of the deformation parameter  $\theta^{IJ}$ , the leading term being the ordinary commutative CS action in 5D (2). Unlike the case of CS in 3D, which is invariant under Seiberg-Witten map, the 5D case exhibits a nontrivial first order NC correction that has been computed in [3].

Finally, as we seem to be living in four dimensional space-time, one ought to dimensionally reduce CS action on  $S^1$  to get a phenomenologically acceptable theory. It was shown in [2] that action (2), dimensionally reduced, yields

$$L \sim \varepsilon_{ABCDE} \phi^A F^{BC} F^{DE}, \quad (4)$$

for a suitably defined field  $\phi^A$ . Action (4) has  $SO(3,2)$  gauge symmetry, that we break down to its Lorentz part by choosing  $\phi^A = (0, 0, 0, 0, l)$ , as in [4], and we are left with just Einstein-Hilbert Lagrangian with a cosmological constant. In our work, we consider this scenario in a noncommutative theory, where in addition to noncommutativity in the usual four dimensions there is a noncommutativity between those four dimensions and the compactified fourth spatial dimension<sup>2</sup>. We concentrate only on the gravity part and obtain the first-order correction to the Lagrangian (4). After symmetry breaking, and under suitable assumptions about the vector fields  $X_A$ , we are left with only four different terms in the first-order correction, that are given by:

$$\sim \theta^{IJ} X_I^\alpha X_J^A \varepsilon^{\mu\nu\rho\sigma} \left( \frac{1}{2l^4} R_{\mu\nu}^{ab} T_{\rho\sigma a} e_{ab} - \frac{2}{l^4} T_{\mu\nu}^a R_{\alpha\rho ab} e_\sigma^b + \frac{1}{l^4} R_{\mu\nu}^{ab} T_{\alpha\rho a} e_{\sigma b} + \frac{3}{l^6} T_{\mu\nu}^a e_{\rho a} g_{\alpha\sigma} \right) \quad (5)$$

Varying this action with respect to the spin connection and tetrad fields, one obtains corrections to the equation of motions. We then proceed to analyse solutions to those equations. For example, AdS solution is a solution of those equations even after including the first order correction. This work is still in progress.

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<sup>2</sup>Without this assumption, there is no first order correction, as was the case in [4]

# THE ENTROPY OF HAWKING RADIATION IN 2D DILATON GRAVITY

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## 1 Introduction

During seventies, Hawking discovered that black holes emit radiation. That was the first theoretical proof that black holes can be understood as thermal objects. But the problem of unitary evolution of Hawking radiation remained unanswered for next few decades. Even though Don Page came up with the idea of a curve that entropy should follow if evolution was unitary, that curve hasn't been reproduced in any gravitational model for a little less than thirty years. Using gravitational path integral, a new formula for gravitational fine-grained entropy was derived in early 2010s:

$$S_{FG} = \min_I \left\{ \text{ext}_I \left[ \frac{A[I]}{4G} + S_{\text{semi-cl}}(\Sigma_I \cup \Sigma_{\text{rad}}) \right] \right\}. \quad (1)$$

With help of this formula, Page's curve has been reconstructed in 2D dilaton gravity models. First in *JT* gravity in 2019, and than in *RST/BPP*[3] models in 2021. We will briefly explain how this new formula is derived and how it works, as is explained in [1], and than we will use it to reproduce the results from [3]. Than we will go over a new model of dimensionally reduced Einstein-Hilbert action:

$$S = \frac{1}{4G} \int d^2x \sqrt{-g} e^{-2\phi} [\mathcal{R} + 2(\nabla\phi)^2 + 2\lambda^2 e^{2\phi}] - \frac{1}{2} \sum_{i=1}^N \int d^2x \sqrt{-g} (\nabla f_i)^2, \quad (2)$$

and reproduce Page curve in this model for the scenario of eternal black hole.

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## TIME REPARAMETRIZATION AND INTEGRABILITY OF CHAPLYGIN SYSTEMS

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The nonholonomic systems are not Hamiltonian. Moreover, in general, they do not preserve an invariant measure. We consider a class of systems called  $G$ -Chaplygin system. They are defined as nonholonomic systems that are  $G$ -invariant. More precisely, the Lie group  $G$  acts freely on the configuration space  $Q$  and the quotient space  $N = Q/G$  is a manifold, the Lagrangian  $L$  and the nonintegrable distribution of constraints  $\mathcal{D}$  are  $G$ -invariant, such that  $\mathcal{D}$  is a principal connection of the bundle  $Q \rightarrow N = M/G$ . Then the system reduces to the tangent bundle of the base manifold  $N$ . In the case of the existence of an invariant measure, Chaplygin proposed a remarkable Hamiltonization procedure of reduced equations, introducing time reparametrization. This procedure is called Chaplygin multiplier method.

We study a time reparametrisation of the Newton type equations on Riemannian manifolds slightly modifying the Chaplygin multiplier method, allowing us to consider the Chaplygin method and the Maupertuis principle within a unified framework. We present the non-holonomic problem of rolling without slipping and twisting of a balanced ball over a fixed sphere in  $\mathbf{R}^n$ . We prove that the problem always has an invariant measure. It appears that this is a  $SO(n)$ -Chaplygin system that reduces to the cotangent bundle  $T^*S^{n-1}$ . Two integrable cases are presented. The first one is obtained for a special inertia operator that allows the Chaplygin Hamiltonization of the reduced system. In the second case, we consider the rigid body inertia operator  $\mathcal{S}\omega = I\omega + \omega I$ ,  $I = \text{diag}(I_1, \dots, I_n)$  with a symmetry  $I_1 = I_2 = \dots = I_r \neq I_{r+1} = I_{r+2} = \dots = I_n$ . It is proved that the general trajectories are quasi-periodic, while for  $r \neq 1, n-1$  the Chaplygin reducing multiplier method does not apply.

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## QUANTUM VERIFICATION WITH LIMITED RESOURCES

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Entangled states are one of the indispensable ingredients of almost all quantum technologies. As a resource, entanglement is an essential tool for quantum communication, computation, cryptography and quantum sensing. Harvesting full quantum advantage demands an increased precision in manufacturing the components and devices. To achieve this, one needs to develop efficient and reliable certification techniques. There are several reasons why verification is necessary. In principle, the sources of quantum states are rarely perfect, due to unavoidable errors, noise or decoherence. In some cases, the purchased sources are not trusted, and before their usage, they need to be characterised or validated.

The efficiency of a certification technique is determined by its complexity. The reliability of a technique is related to the confidence with which it can certify the presence of a certain state or some of its properties. An equally important aspect of a certification technique is its informativeness: how much it tells us about the underlying state. This depends on the application, as for some tasks it is enough to certify the presence of some quantum resource, and sometimes the full description of the state is needed. In the intermediate scenario, one is interested in some kind of hypothesis testing: whether the underlying state is within some predetermined distance from the target state. There are different aspects of complexity, but it is always intertwined with the desired reliability and informativeness. Here, we focus on the complexity which can be characterized by the number of different measurements one needs to apply to certify a certain quantum state (measurement complexity) and by the total number of the copies of the state, needed to achieve particular information with given reliability (sample complexity). A compact comparison of different techniques in terms of complexity can be found in [1], as well as in [2, 3] two recent reviews of the certification techniques.

The special case of quantum verification is the scenario with uncharacterised measurements, the so-called black-box scenario, in which practical verification methods are still rather scarce. The development of self-testing methods is an important step forward, but these results so far have been used for reliable verification only by considering the asymptotic behaviour of large, identically and independently distributed (IID) samples of a quantum resource. Such strong assumptions deprive the verification procedure of its truly device-independent character.

In our recent work [4], we developed a systematic approach to device-independent verification of quantum states free of IID assumptions in the finite copy regime by combining device-independent quantum state verification with self-testing. We showed that the developed protocol can be performed with optimal sample efficiency. Furthermore, we developed

a device-independent protocol for quantum state certification, in which a fragment of the resource copies is measured to warrant the rest of the copies to be close to some target state.

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## EVOLUTION OF QUANTUM CORRELATIONS IN GAUSSIAN NOISY CHANNELS

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We describe the behaviour of continuous variable quantum correlations (entanglement, Gaussian quantum discord, Gaussian quantum steering) and quantum coherence in a system of two (coupled or uncoupled) bosonic modes evolving in a Gaussian noisy channel, in the case of a common environment of the form of a thermal bath or a squeezed thermal bath. We solve the Markovian master equation for the time evolution of the considered system and study the quantum correlations and quantum coherence in terms of covariance matrices for Gaussian input states (squeezed vacuum state and squeezed thermal state). Depending on the initial state of the system, the coefficients describing the interaction of the system with the reservoir and the intensity of the coupling between the two modes, we observe phenomena like generation, suppression, periodic revivals and suppressions, or an asymptotic decay in time of quantum correlations and relative entropy of coherence [1, 2, 3, 4, 5].

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## DO WE UNDERSTAND THE UNIVERSE?

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### 1 Summary and Conclusions

The standard,  $\Lambda$ CDM, cosmological model, has successfully passed increased scrutiny, as observations of the cosmic microwave background (CMB), type- Ia supernovae (SNeIa) and large-scale structure have improved drastically over recent years. Nonetheless, tensions have arisen for specific parameters when their values are inferred, within the  $\Lambda$ CDM, from different probes and observables. In this talk I will review, what, if any, evidence we have that the  $\Lambda$ CDM has broken down and if hints of new physics are already in the data. I will describe the way forward to firm up these hints.

The discrepancies between model-independent measurements and model-dependent inferred values of  $H_0$  from different experiments (each of them sensitive to different physics and systematic errors) might be a hint for the need of modifying the standard  $\Lambda$ CDM model. The most promising deviations from  $\Lambda$ CDM proposed to solve such tensions involve a boost in the expansion rate before recombination, as to lower the value of  $r_d$  and reconcile the direct and the inverse distance ladder. However, we argue in this work, there is a more varied phenomenology, that goes well beyond  $r_d$ , to be matched by any new physics put forward to solve the  $H_0$  tension, especially regarding cosmic ages: the trouble goes beyond  $H_0$ .

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# GRAVITON MASS BOUNDS FROM STELLAR ORBITS AROUND THE GALACTIC CENTER

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## 1 Introduction

Here we consider some phenomenological consequences of Yukawa form of massive gravity and its applications for obtaining the graviton mass bounds from analysis of the bright star orbits around the central supermassive black hole (SMBH) of our Galaxy. We present a short overview of the most important results of our investigations in which we considered the following Yukawa-like modification of the Newtonian gravitational potential in the weak field approximation [1]:

$$\Phi(r) = -\frac{GM}{(1+\delta)r} \left( 1 + \delta e^{-\frac{r}{\Lambda}} \right), \quad (1)$$

where  $\Lambda$  is the range of Yukawa interaction and  $\delta$  is a universal constant.

We first performed two-body simulations of the stellar orbits in the gravitational potential (1) and then the obtained results were compared with the observed orbit of S2 star around the Galactic Center (see [1] for more details). This enabled us to constrain the range of Yukawa interaction  $\Lambda$ , which was found to be on the order of several thousand astronomical units (AU). Assuming that this parameter corresponds to the Compton wavelength of graviton, we estimated the upper bound for graviton mass to  $m_g < 2.9 \times 10^{-21}$  eV, which is consistent with the corresponding LIGO results, but obtained in an independent way [2]. We also studied the

possibility to improve this estimate with future observational facilities, assuming that they will confirm the prediction of General Relativity (GR) for orbital precession [3]. For that purpose we analyzed the case when Yukawa gravity induced the same orbital precession as GR, and concluded that the future observations of the bright stars near the Galactic Center could provide an opportunity to constrain the upper bound for graviton mass to  $m_g < 5 \times 10^{-23}$  eV, which is slightly better than the current estimates obtained by LIGO and close to the predicted estimates of the forthcoming gravitational wave interferometers and pulsar timing observations [3]. Moreover, we studied the possible influence of the bulk distribution of matter which includes stellar cluster, interstellar gas and dark matter near SMBH and found that for larger mass density distributions of the extended matter, the corresponding estimates for graviton mass could be slightly larger but still in the expected interval [4].

## 2 Conclusion

We demonstrated that the observations of the stellar orbits around the Galactic Center and their analysis in the frame of the massive gravity theories represent a very efficient and powerful tool for studying the fundamental gravitational physics, in particular for constraining the graviton mass, evaluating the gravitational potential at the Galactic Center and testing the GR predictions.

Since 2019, our estimate for graviton mass upper bound of  $m_g < 2.9 \times 10^{-21}$  eV is included in the *Gauge and Higgs Boson Particle Listings* published by the Particle Data Group (PDG) (see Ref. ZAKHAROV 16 in [5]).

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## NON-SUPERSYMMETRIC STRING PHENOMENOLOGY

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We study 4d heterotic string compactifications with Pati-Salam gauge group exhibiting spontaneous supersymmetry breaking via the Scherk-Schwarz mechanism associated with two large internal dimensions.

We identify an interesting class of super-no-scale chiral models that address some central issues in nonsupersymmetric string phenomenology including the elimination of tachyons and the suppression of the cosmological constant.

## A NONLOCAL GRAVITY MODEL AND ITS EXACT COSMOLOGICAL SOLUTIONS

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We consider modification of general relativity extending  $R - 2\Lambda$  by nonlocal term of the form  $\sqrt{R - 2\Lambda} \mathcal{F}(\square)\sqrt{R - 2\Lambda}$ , where  $\mathcal{F}(\square)$  is an analytic function of the d'Alembert operator  $\square$ . We have found some exact cosmological solutions of the corresponding equations of motion without matter and with  $\Lambda \neq 0$ . One of these solutions is  $a(t) = A t^{\frac{2}{3}} e^{\frac{\Lambda}{14} t^2}$ , which imitates properties similar to an interplay of the dark matter and the dark energy. For this solution we calculated some cosmological parameters which are in a good agreement with observations.

This is joint work with I. Dimitrijevic, B. Dragovich, A. S. Koshelev and Z. Rakic.

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# QUANTUM GRAVITY AND ELEMENTARY PARTICLES FROM HIGHER GAUGE THEORY

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The higher category theory can be employed to generalize the  $BF$  action to the so-called  $nBF$  action, by passing from the notion of a gauge group to the notion of a gauge  $n$ -group. The novel algebraic structures called 2-groups, 3-groups and 4-groups are designed to extend the notions of connection and parallel transport across higher dimensional manifolds. They generalize the concept of gauge symmetry, giving rise to a class of topological actions called  $2BF$ ,  $3BF$  and  $4BF$  actions.

We can employ these topological actions by adding appropriate simplicity constraints, in order to describe the correct dynamics of Yang-Mills, Klein-Gordon, Dirac, Weyl and Majorana fields coupled to Einstein-Cartan gravity. It is straightforward to rewrite the whole Standard Model coupled to gravity as a constrained  $3BF$  or  $4BF$  action. The split into a topological sector and simplicity constraints is adapted to the spinfoam quantization techniques, giving rise to a full model of quantum gravity with matter.

In addition, the properties of the gauge  $n$ -group structure opens up a possibility of a nontrivial unification of all fields. An  $n$ -group naturally contains novel gauge groups which specify the spectrum of matter fields present in the theory, just like the ordinary gauge group specifies the spectrum of gauge bosons in the Yang-Mills theory. The presence and the properties of these new gauge groups has the potential to possibly explain fermion families, and other structure in the matter spectrum of the theory.

Based on [1, 2, 3, 4, 5].

## References

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**CROSS DISCIPLINARY SCIENCE AND ART  
ENGAGEMENT AND NETWORKING**

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**COSMOLOGY AND NEW PHYSICS FROM  
LARGE-SCALE STRUCTURE OF THE UNIVERSE**

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## Program

**Venue:** Faculty of Physics, Studentski Trg 12, Belgrade

**Room:** Hall 661

### Tuesday, 7 September 2021

Arrival and Registration

### Wednesday, 8 September 2021

**09:00 – 11:00** BPU – SEENET Meeting – PhD students' program

**Venue:** Serbian Academy Of Sciences and Arts (SASA), Hall 3

**15:00 – 17:00 Session I**

**R. Jimenez**, *Do we understand the Universe?*

**I. Rizos**, *Non-supersymmetric string phenomenology*

**M. Simonovic**, *Cosmology and new physics from large-scale structure of the universe*

**17:00 – 17:30 Coffee Break**

**17:30 – 19:00 Session II**

**A. Isar**, *Evolution of quantum correlations in Gaussian noisy channels*

**E. M. Cioroianu**, *Jacobi-like structures in the line bundle language*

**M. Hoch**, *Cross disciplinary science and art engagement and networking*

### Thursday, 9 September 2021

**09:00 – 10:30 Session III**

**A. Gocanin**, *Quantum verification with limited resources*

**B. Gajic**, *Time reparametrization and integrability of Chaplygin systems*

**M. Dimitrijevic-Ciric**, *L-infinity algebra and braided field theory*

**10:30 – 11:00 Coffee Break**

**11:00 – 12:30 Session IV**

**P. Jovanovic**, *Graviton mass bounds from stellar orbits around the Galactic Center*

**D.D. Dimitrijevic**, *About tachyon inflation in the holographic braneworld*

**M. Vojinovic**, *Quantum gravity and elementary particles from higher gauge theory*

**12:30 – 15:00 Lunch Break**

**15:00 – 16:30 Session V**

**B. Cvetkovic**, *Entropy of Reisner-Nordstrom like black holes*

**S. Djordjevic**, *The Entropy of Hawking Radiation in 2D Dilaton Gravity*

**D. Djordjevic**, *4D NC Gravity from NC 5D CS Theory*

**16:30 – 17:00 Coffee Break**

**17:00 – 17:30**

**T. Vetsov**, *Thermodynamics and complexity of holographic backgrounds*

**17:30 – 18:30 Discussion**

Closing

**Friday, 10 September 2021**

Departure

## Program Committee

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