# About Tachyon Inflation in the Holographic Braneworld

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# Introduction

- We study tachyon inflation in the framework of holographic cosmology.
- In the study of the holographic braneworld an important property of an asymptotically AdS bulk is that AdS space is dual to a conformal field theory at its boundary.
- This model is based on a holographic braneworld scenario with an effective tachyon field on a D3-brane located at the holographic boundary of an asymptotic $ADS_5$ bulk.

# Holographic cosmology

- We assume the holographic braneworld is a spatially flat FRW universe

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t)(dr^2 + r^2 d\Omega^2)$$

- and use holographic Friedmann equations

$$H^2 = \frac{8\pi G_\mathrm{N}}{3}\rho \rightarrow H^2 - \frac{\ell^2}{4}H^4 = \frac{8\pi G_\mathrm{N}}{3}\rho$$

$$\dot{H} + H^2 = -4\pi G_\mathrm{N}(\frac{1}{3}p + \rho) \rightarrow \dot{H}\left(1 - \frac{\ell^2}{2}H^2\right) = -4\pi G_\mathrm{N}(p + \rho)$$

# Holographic cosmology

- From holographic cosmology:

$$H^2 = \frac{2}{\ell^2}\left(1 \pm \sqrt{1 - \frac{8\pi G_\mathrm{N}}{3}\ell^2 \rho}\right)$$

- We do not want modified cosmology to depart too much from the standard cosmology after the inflation era, so we demand that above equation reduces to the standard Friedmann equation in the low density limit:

$$G_\mathrm{N}\ell^2\rho \ll 1$$

- This happens only with the minus sign solution.

## Holographic cosmology

- At the same time it follows that the physical range of the Hubble rate is between zero and the maximal value

$$H_{\max} = \frac{\sqrt{2}}{\ell}$$

- This corresponds to the maximal value of the energy density

$$\rho_{\max} = \frac{3}{8\pi G \ell^2}$$

## Holographic cosmology

- Assuming no violation of the strong energy condition, the expansion rate is a monotonously decreasing function of time.
- The universe evolution starts from $t = 0$ with an initial

$$H_{\mathrm{i}} \leq H_{\mathrm{max}}$$

- Energy density and cosmological scale are both finite.
- In this type of (modified gravity) models the Big Bang singularity is avoided.

# Holographic cosmology

- Tachyon matter in the holographic braneworld is described by the Lagrangian

$$\mathcal{L} = -\ell^{-4} V(\theta / \ell)\sqrt{1 - g^{\mu\nu}\theta_{,\mu}\theta_{,\nu}}$$

- Hamiltonian:

$$\mathcal{H} = \ell^{-4} V \sqrt{1+\eta^2}$$

$$\eta = \frac{\sqrt{g_{\mu\nu}\pi^{\mu}\pi^{\nu}}}{\ell^4 V}$$

## Holographic cosmology

- EoMs:

$$\dot{\theta} = \frac{\eta}{\sqrt{1+\eta^2}}$$

$$\dot{\eta} = -3H\eta - \frac{V_{,\theta}}{V}\left(\sqrt{1+\eta^2} + \frac{\eta^2}{\sqrt{1+\eta^2}}\right)$$

- As usual, the Lagrangian and Hamiltonian are identified with the pressure and energy density, respectively

$$p \equiv \mathcal{L}$$

$$\rho \equiv \mathcal{H}$$

# Attractor behavior

- The term attractor is related to the specific properties of the solutions in the phase space. Although the properties of the attractors depend on the choice of coordinates, the phase space we are going to work with will be represented as $(\theta, \dot{\theta})$ plane, as it is usually done.
- Starting from the classical background and spatially homogenous tachyon field, the equation of motion in the phase plane has the general form

$$\frac{d\dot{\theta}}{d\theta} = g(\theta, \dot{\theta})$$

## Attractor behavior

$$\frac{d\dot{\theta}}{d\theta} = g(\theta, \dot{\theta})$$

$$g(\theta, \dot{\theta}) = -\frac{1}{\dot{\theta} p_{,\dot{\theta}\dot{\theta}}} \left[ 3\frac{h}{\ell} p_{,\dot{\theta}} + \dot{\theta} p_{,\dot{\theta}\theta} - p_{,\theta} \right],$$

- This expression is defining equation for the phase space trajectory of the tachyon field.
- The attractor trajectory is defined as

$$\frac{d\dot{\theta}}{d\theta} \simeq 0$$

# Attractor behavior

- or, equivalently

$$g(\theta,\dot{\theta}) \simeq 0$$

- Tachyon case:

$$-\frac{1}{\dot{\theta}}\left[3H\dot{\theta}+\frac{V_{,\theta}}{V}\right] \simeq 0$$

- Attractor behavior of the solution for general tachyonic potential leads to the expression for deviation of the expansion rate in the form

$$\delta H(\theta) = \delta H(\theta_{\mathrm{i}})e^{-3N}$$

# Attractor behavior

$$\delta h(\theta) = \delta h(\theta_{\rm i}) e^{-3N}$$

$$N \equiv \int_{t_{\rm i}}^{t_{\rm f}} H\,dt$$

- Regardless of the initial condition, the attractor behaviour implies that late-time solutions are the same up to a constant time shift, which cannot be measured.

# Inflation

- Slow evolution of the tachyon field with the slow-roll (SR) conditions:

$$\dot{\theta}^2 \ll 1$$

$$|\ \ddot{\theta}\ | \ll 3H\dot{\theta}$$

- It may be shown that the SR conditions are equivalent to

$$\dot{\theta} \simeq \eta \ll 1$$

$$|\ \dot{\eta}\ | \ll 3H\eta$$

# Inflation

- Then, during inflation in the SR regime we have:

$$h^2 \simeq 2(1 - \sqrt{1 - \frac{\kappa^2 V}{3}})$$

- Dimensionless expansion rate:

$$h \equiv \ell H$$

- Dimensionless coupling:

$$\kappa^2 = \frac{8\pi G_{\mathrm{N}}}{\ell^2}$$

## Inflation

- From the EoMs in the SR regime:

$$\dot{\theta} \simeq -\frac{\ell V_{,\theta}}{3Vh}$$

$$\ddot{\theta} \simeq \frac{\ell V_{,\theta}\dot{h}}{3Vh^2} + \left[\left(\frac{V_{,\theta}}{V}\right)^2 - \frac{V_{,\theta\theta}}{V}\right]\frac{\ell\dot{\theta}}{3h}$$

- As mentioned before, the evolution is constrained by the physical range of the expansion rate:

$$0 \leq h^2 \leq 2$$

# Inflation

- The first two SR parameters:

$$\varepsilon_1 \equiv -\frac{\dot{H}}{H^2} = \frac{4-h^2}{12h^2(2-h^2)}\left(\frac{\ell V_{,\theta}}{V}\right)^2$$

$$\varepsilon_2 \equiv \frac{\dot{\varepsilon}_1}{H\varepsilon_1} = 2\varepsilon_1\left(1-\frac{2h^2}{(2-h^2)(4-h^2)}\right)+\frac{2\ell^2}{3h^2}\left[\left(\frac{\ell V_{,\theta}}{V}\right)^2-\frac{\ell^2 V_{,\theta\theta}}{V}\right]$$

- During inflation both parameters are less then 1 and inflation ends once either of the two exceeds unity.
- The third slow-roll parameter:

# Inflation

$$\varepsilon_3 \equiv \frac{\dot{\varepsilon}_2}{H\varepsilon_2} = \varepsilon_2 + \frac{4h^2(8-h^4)}{(2-h^2)(4-h^2)(8-8h^2+h^4)}\varepsilon_1$$

- In the following we will study a simple exponential potential:

$$V = e^{-\omega\theta/\ell}$$

- Arbitrary initial value of the field: $-\infty \leq \theta_i \leq \infty$
- For this potential we have

$$\left(\frac{\ell V_{,\theta}}{V}\right)^2 = \frac{\ell^2 V_{,\theta\theta}}{V} = \omega^2$$

## Inflation

- EoM with the exponential potential may be easily integrated yielding the time as a function $h$ in the SR regime:

$$t = \frac{3\ell}{\omega^2}\left[2(\sqrt{2}-h) + \ln\frac{(\sqrt{2}-1)(2+h)}{(\sqrt{2}+1)(2-h)}\right]$$

- Number of e-folds:

$$N \equiv \int_{t_\mathrm{i}}^{t_\mathrm{f}} H dt \simeq -3\int_{\theta_\mathrm{i}}^{\theta_\mathrm{f}} \frac{Vh^2}{\ell^2 V_{,\theta}} d\theta$$

# Inflation

- Number of e-folds:

$$N = \frac{12}{\omega^2}\left[\sqrt{1-\frac{\omega^2}{3}} - 1 + \frac{h_i^2}{2} + \ln\left(2-\frac{h_i^2}{2}\right) - \ln\left(1+\sqrt{1-\frac{\omega^2}{3}}\right)\right]$$

- Approximate expression (up to $\omega^2$ ):

$$N = \frac{12}{\omega^2}\left[\frac{h_i^2}{2} + \ln\left(1-\frac{h_i^2}{4}\right)\right] - 1$$

- In the SR regime neither $N$ nor the slow-roll parameters depend on $\kappa$ .

# Inflation

- Adiabatic sound speed:

$$c_{\mathrm{s}}^2 \equiv \left.\frac{\partial p}{\partial \rho}\right|_{\theta} = 1 - \frac{4-2h^2}{12-3h^2}\varepsilon_1$$

- Tensor-to-scalar ratio:

$$r \equiv \frac{\mathcal{P}_{\mathrm{T}}}{\mathcal{P}_{\mathrm{S}}}$$

- Scalar spectral index:

$$n_{\mathrm{s}} \equiv \left.\frac{d \ln \mathcal{P}_{\mathrm{S}}}{d \ln q}\right|_{q=aHc_s^{-1}}$$

# Inflation

- At the lowest order in the SR parameters:

$$\mathcal{P}_{\mathrm{T}} \simeq \frac{2\kappa^2 h^2}{\pi^2}[1 - 2(1 + C)\varepsilon_1]$$

$$\mathcal{P}_{\mathrm{S}} \simeq \frac{\kappa^2 h^2}{8\pi^2(1 - h^2 / 2)c_{\mathrm{s}}\varepsilon_1} \times \left[1 - 2\left(1 + C + \frac{Ch^2}{2 - h^2}\right)\varepsilon_1 - C\varepsilon_2\right]$$

$$C = -2 + \ln 2 + \gamma \simeq -0.72$$

# Inflation

- Tensor-to-scalar ratio:

$$r = 8(2-h^2)\varepsilon_1\left[1 + C\varepsilon_2 + 2\left(\frac{Ch^2}{2-h^2} - \frac{2-h^2}{12-3h^2}\right)\varepsilon_1\right]$$

- Scalar spectral index:

$$n_{\mathrm{s}} = 1 - \left(2 + \frac{2h^2}{2-h^2}\right)\varepsilon_1 - \varepsilon_2 - \left(2 + \frac{2h^2}{2-h^2} - \frac{8h^2}{3(4-h^2)^2} - \frac{8Ch^2}{(2-h^2)^2}\right)\varepsilon_1^2 - \left(\frac{8}{3} + \frac{h^2}{3(4-h^2)} + \frac{4C}{2-h^2}\right)\varepsilon_1\varepsilon_2 - C\varepsilon_2\varepsilon_3$$

# Inflation

- Comparison with the predictions of the standard tachyon inflation (identical to the predictions of the canonical scalar inflation at the first order in the SR parameters) shows a substantial deviation:

$$r = 8(2-h^2)\varepsilon_1\left[1 + C\varepsilon_2 + 2\left(\frac{Ch^2}{2-h^2} - \frac{2-h^2}{12-3h^2}\right)\varepsilon_1\right]$$

$$r\,|_{\mathrm{st}} = 16\varepsilon_1\left(1 - C\varepsilon_2 - \frac{1}{3}\varepsilon_1\right)$$

# Inflation

$$n_{\mathrm{s}} = 1 - \left(2 + \frac{2h^2}{2-h^2}\right)\varepsilon_1 - \varepsilon_2 - \left(2 + \frac{2h^2}{2-h^2} - \frac{8h^2}{3(4-h^2)^2} - \frac{8Ch^2}{(2-h^2)^2}\right)\varepsilon_1^2 - \left(\frac{8}{3} + \frac{h^2}{3(4-h^2)} + \frac{4C}{2-h^2}\right)\varepsilon_1\varepsilon_2 - C\varepsilon_2\varepsilon_3$$

$$n_{\mathrm{s}}\,\big|_{\mathrm{st}} = 1 - 2\varepsilon_1 - \varepsilon_2 - 2\varepsilon_1^2 - \left(\frac{8}{3} + 2C\right)\varepsilon_1\varepsilon_2 - C\varepsilon_2\varepsilon_3$$

# Final remarks

- We have investigated a model of tachyon inflation based on a holographic braneworld scenario with a brane located at the boundary of the AdS5 bulk.
- The model posesses attractor behavior.
- The SR equations in this model turn out to differ substantially from those of the standard tachyon inflation with the same potential.
- In the SR approximation and for given number of e-folds our results depend only on the initial value of the Hubble rate and do not depend on the fundamental coupling.
- Agreement with observations is better for larger $N$.

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THANK YOU!

ХВАЛА!

## AdS/CFT and holographic cosmology

- General asymptotically AdS5 metric in FG coordinates:

$$ds^2 = G_{ab}dx^a dx^b = \frac{\ell^2}{z^2}\left(g_{\mu\nu}dx^\mu dx^\nu - dz^2\right)$$

- AdS curvature radius: $\ell$
- 5-dim bulk action in AdS5 background:

$$S_{(5)}[\Phi] = \int d^5x\sqrt{G}\mathcal{L}_{(5)}(\Phi, G_{ab})$$

- Induced metric on the boundary: $h_{\mu\nu}$
- Boundary value of the bulk field $\Phi$: $\Phi(x, z=0) \equiv \phi(x)$

## AdS/CFT and holographic cosmology

- Define a functional:

  $$S[\phi, h] = S^{\mathrm{sh}}_{(5)}\left[\Phi[\phi, h]\right]$$

- The AdS/CFT conjecture states that $S[\phi, h]$ can be identified with the generating functional of a conformal field theory on the boundary.
- The boundary field serves as a source for CFT operators.
- In this way the CFT correlation functions can be calculated as functional derivatives of the on-shell bulk action.

# AdS/CFT and holographic cosmology

- Similarly, the induced metric serves as the source for the stress tensor operator of the dual CFT so that its VEV is obtained as:

$$\frac{1}{2\sqrt{|h|}}\frac{\delta S_{(5)}}{\delta h^{\mu\nu}} = \langle T_{\mu\nu}^{\mathrm{CFT}} \rangle$$

## AdS/CFT and holographic cosmology

- Consider the bulk action with only gravity in the bulk:

$$S_{(5)}[G] = \frac{1}{8\pi G_5}\int d^5x\sqrt{G}\left[-\frac{R^{(5)}[G]}{2} - \Lambda_5\right]$$

$$\Lambda_5 = -6/\ell^2$$

- The on-shell action is IR divergent and need to be regularized and renormalized.
- The asymptotically AdS metric near $z = 0$ can be expanded as

$$g_{\mu\nu}(z,x) = g^{(0)}_{\mu\nu}(x) + z^2 g^{(2)}_{\mu\nu}(x) + z^4 g^{(4)}_{\mu\nu}(x) + \cdots$$

# AdS/CFT and holographic cosmology

- Regularize the action by placing a brane near the boundary
  $z = \epsilon\ell, \quad \epsilon \ll 1$
- Induced metric on the brane is

$$h_{\mu\nu} = \frac{1}{\epsilon^2} g_{\mu\nu}(\epsilon\ell, x) = \frac{1}{\epsilon^2}\left(g^{(0)}_{\mu\nu} + \epsilon^2\ell^2 g^{(2)}_{\mu\nu} + \epsilon^4\ell^4 g^{(4)}_{\mu\nu} + \cdots\right)$$

- The bulk splits in two regions:

$$0 \le z < \epsilon\ell$$

$$\epsilon\ell \le z < \infty$$

## AdS/CFT and holographic cosmology

- The regularized bulk action:

$$S_{(5)}^{\text{reg}}[h] = \frac{1}{8\pi G_5} \int_{z \geq \epsilon\ell} d^5x \sqrt{G} \left[ -\frac{R^{(5)}}{2} - \Lambda_5 \right] + S_{\text{GH}}[h] + S_{\text{br}}[h]$$

$$S_{\text{br}}[h] = \int d^4x \sqrt{|h|} (-\sigma + \mathcal{L}_{\text{matt}}[h])$$

- The renormalized action is obtained by adding counter-terms and taking the limit $\epsilon \to 0$

$$S^{\text{ren}}[h] = S_{(5)}^{\text{reg}}[h] + S_1[h] + S_2[h] + S_3[h]$$

## AdS/CFT and holographic cosmology

- Variation with respect to the induced metric of the regularized on-shell bulk action vanishes:

$$\delta S_{(5)}^{\text{reg}}[h] = 0$$

- Four-dimensional Einstein's equations on the boundary:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}^{(0)} = 8\pi G_{\text{N}} (\langle T_{\mu\nu}^{\text{CFT}} \rangle + T_{\mu\nu}^{\text{matt}}) \Rightarrow$$

$$H^2 - \frac{\ell^2}{4} H^4 = \frac{8\pi G_{\text{N}}}{3} \rho$$

$$\dot{H}\left(1 - \frac{\ell^2}{2} H^2\right) = -4\pi G_{\text{N}} (p + \rho)$$

# Inflation

- $N = 60$ (dashed line) and $N = 90$ (full line)
- $0 \leq h^2 \leq 2$