Graviton mass bounds from stellar orbits around the Galactic Center

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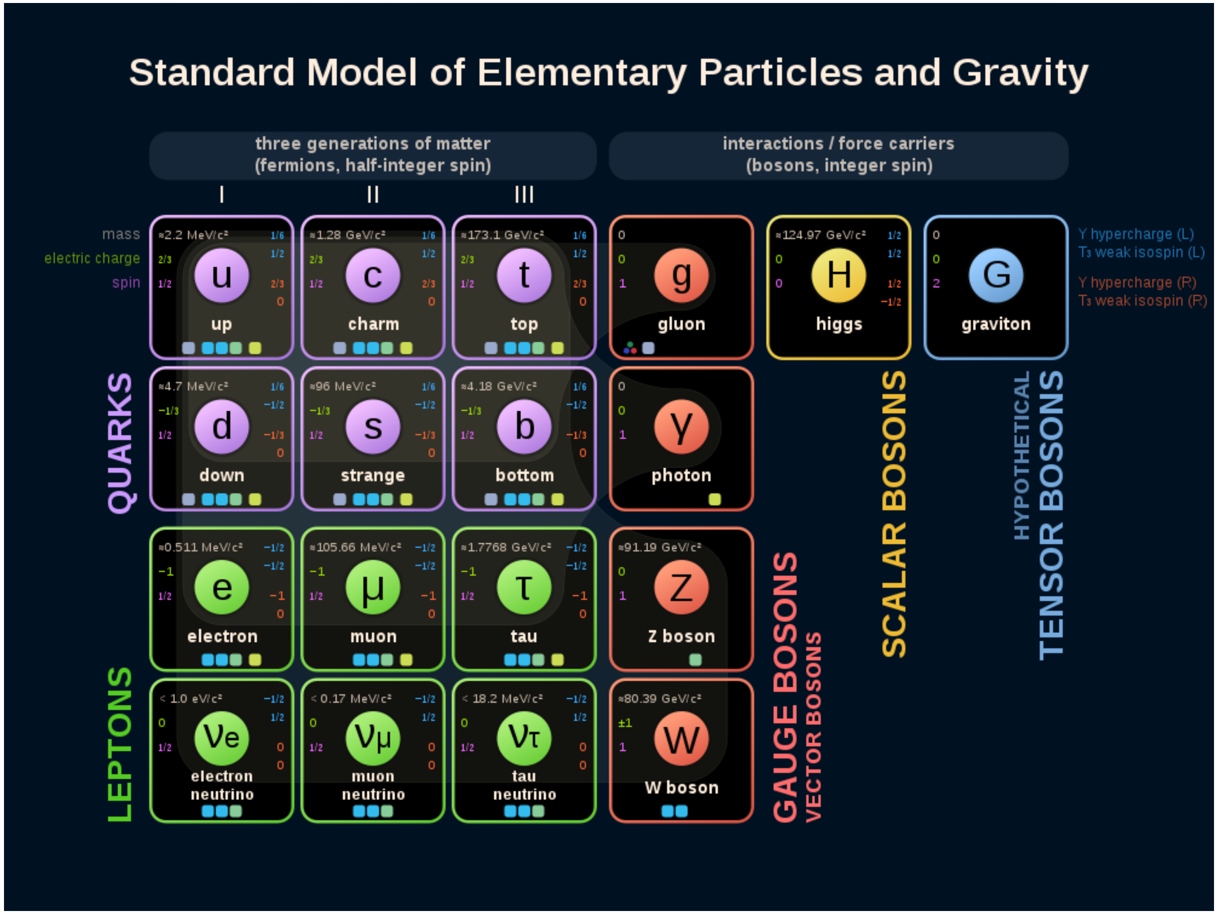
Outline of the talk

- Motivation: addressing the unclear situation with graviton mass
- Yukawa-like version of massive gravity
- Existing constraints on Yukawa gravity and graviton mass
- Observed stellar orbits around Sgr A* at Galactic Center
- Our results: constraining the graviton mass by analysis of the observed stellar orbits in Yukawa gravity
- The results are obtained in collaboration with:
  - Vesna Borka Jovanović and Duško Borka (Serbia)
  - Alexander F. Zakharov (Russia)
  - Salvatore Capozziello (Italia)
- Conclusions

Graviton: gauge boson of gravitational interaction

- Spin: 2 (tensor boson)
- Electric charge: 0 (neutral)
- General Relativity (GR): graviton is massless and travels along null geodesics (like photon), i.e. at the speed of light c
- Theories of massive gravity: gravitation is propagated by a massive field (i.e. by graviton with small, nonzero mass m_g)
- Indroduced in 1939 (Fierz & Pauli, 1939, RSPSA, 173, 211)



- Massive graviton propagates at an energy (or frequency) dependent speed, and the effective Newtonian potential has a Yukawa form (Will, 1998, PRD, 57, 206): $\propto r^{-1}\exp(-r/\lambda_g)$, where $\lambda_g = h/(m_g\, c)$ is the Compton wavelength of graviton
- Modified dispersion relation: $E^2 = p^2c^2 + m_g^2c^4 \quad\Rightarrow\quad v_g^2/c^2 \equiv c^2p^2/E^2 \quad\Rightarrow$

$$\boxed{v_g^2/c^2 = 1 - m_g^2c^4/E^2 = 1 - h^2c^2/(\lambda_g^2E^2) = 1 - c^2/(f\lambda_g)^2}$$

Yukawa gravity

- Gravitational potential with a Yukawa correction can be obtained in the Newtonian limit of any analytic *f*(*R*) gravity model (Capozziello et al. 2014, PRD, 90, 044052)
- Action for *f*(*R*) gravity: $\mathcal{S} = \int d^4x\sqrt{-g}\,[f(R) + \mathcal{X}\mathcal{L}_m]\,, \quad \mathcal{X} = \dfrac{16\pi G}{c^4}$
- 4th-order field equations and their trace:

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - f'(R)_{;\mu\nu} + g_{\mu\nu}\Box f'(R) = \frac{\mathcal{X}}{2}T_{\mu\nu}$$

$$3\Box f'(R) + f'(R)R - 2f(R) = \frac{\mathcal{X}}{2}T$$

- Analytic Taylor expandable function *f*(*R*):

$$f(R) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} R^n = f_0 + f_1 R + \frac{f_2}{2}R^2 + \ldots \quad \Rightarrow$$

- Metric: $ds^2 = \left[1 + \dfrac{2\Phi(r)}{c^2}\right] c^2dt^2 - \left[1 - \dfrac{2\Psi(r)}{c^2}\right] dr^2 - r^2 d\Omega^2$
- Yukawa-like gravitational potential in the weak field limit:

$$\Phi(r) = -\frac{GM}{(1+\delta)r}\left(1 + \delta e^{-\frac{r}{\Lambda}}\right)$$

$$\Lambda^2 = -f_1/f_2 \quad \wedge \quad \delta = f_1 - 1$$

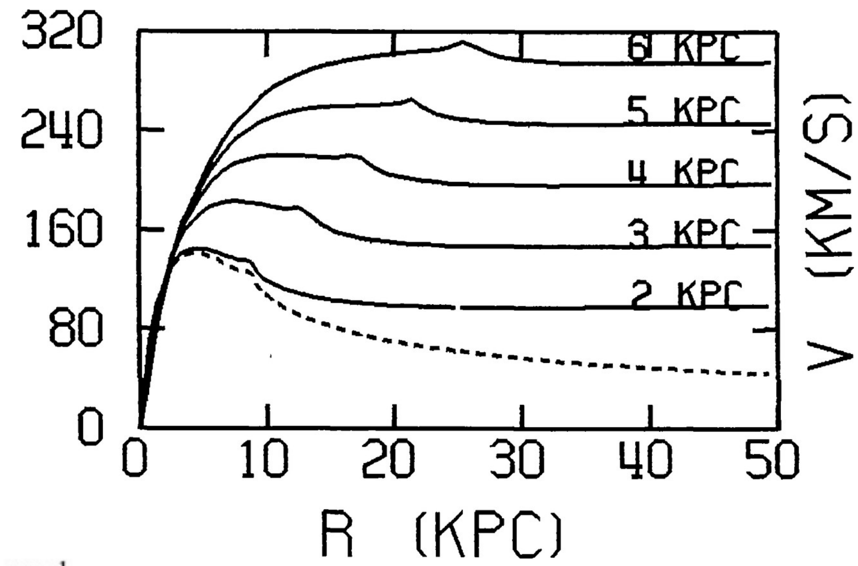
- **Λ - range of Yukawa interaction**
- **δ - universal constant**

Constraints on Yukawa gravity I

- Yukawa-like potential of the form (Sanders, 1984, A&A, 136, L21):

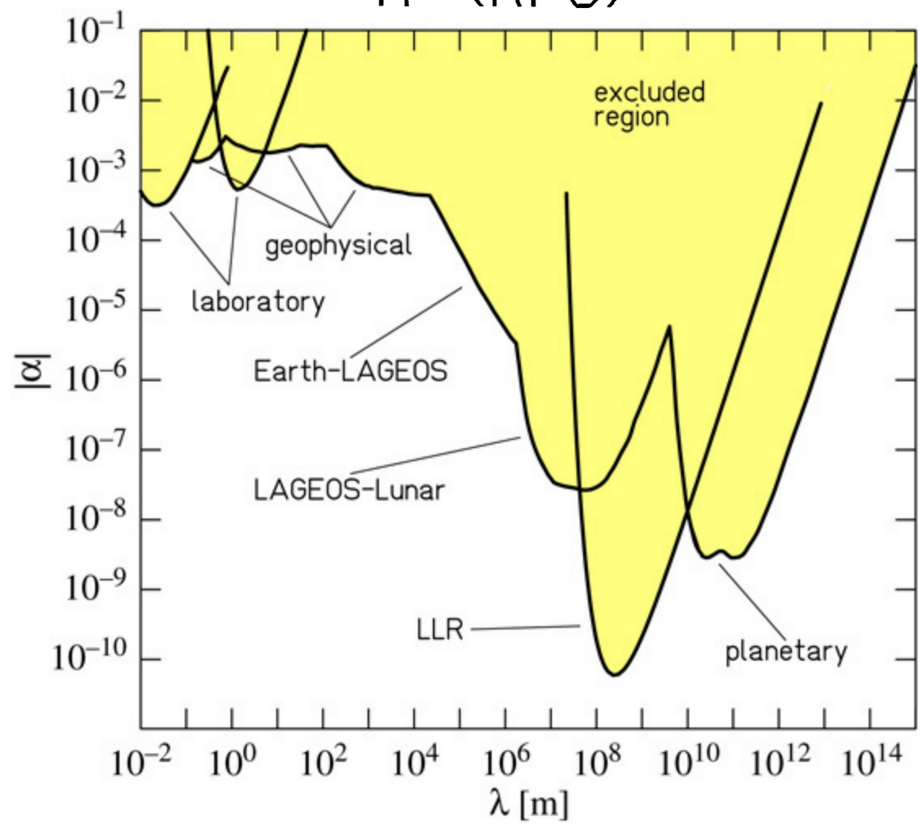
$$U\left(r\right)=\frac{G_{\infty}M}{r}\left(1+\alpha\,e^{-r/r_0}\right),\quad r_0=\frac{h}{m_0c}$$

- Gravitational constant measured locally (G_0) and at infinity (G_∞): $G_0=G_\infty\left(1+\alpha\right)$
- If r_0 corresponds to graviton mass m_0 then flat rotation curves could be accounted for $\alpha \sim -1$
- Additional repulsive (anti-gravity) force



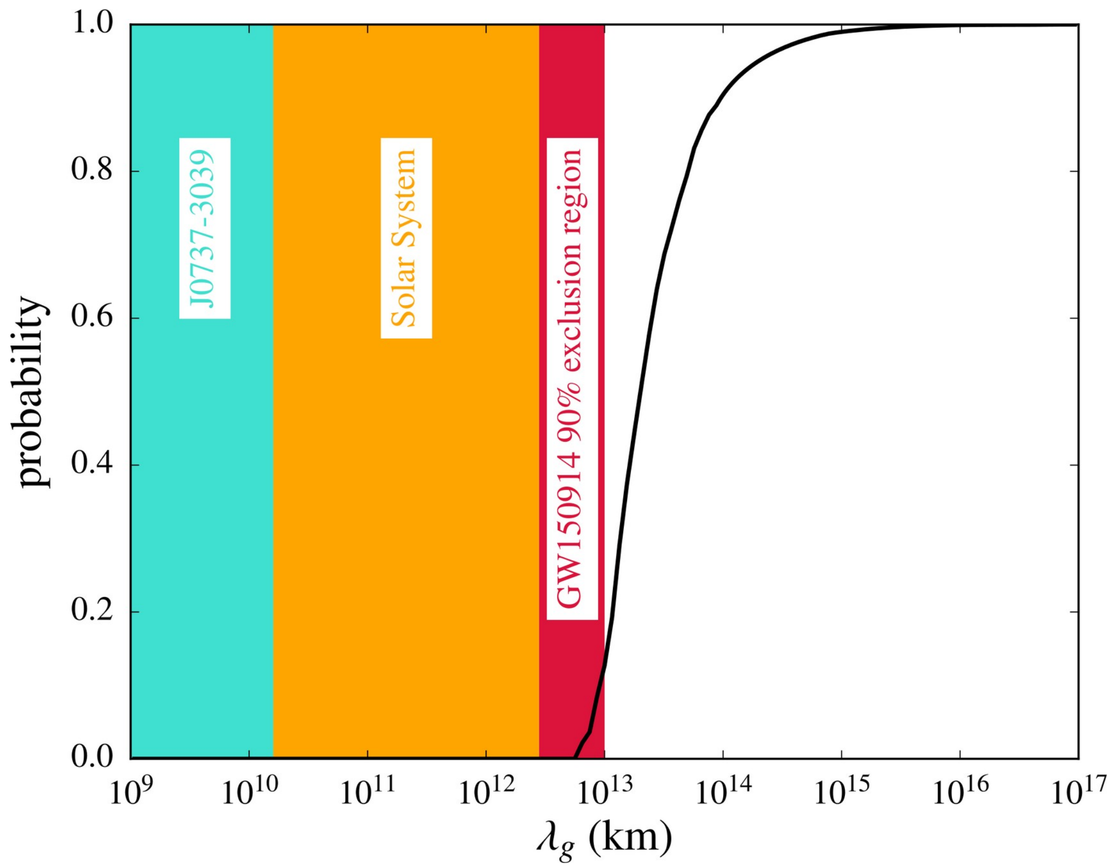
- Experimental constraints on additional Yukawa gravitational interaction between masses m_1 and m_2 (Adelberger et al. 2009, PrPNP, 62, 102):

$$V\left(r\right)=-G_N\frac{m_1\,m_2}{r}\left(1+\alpha\,e^{-r/\lambda}\right)$$



Constraints on Yukawa gravity II

- Probability distribution and exclusion regions for the graviton Compton wavelength λ_g (Abbott et al., LIGO Scientific and Virgo Collaborations, 2016, PRL, 116, 221101)

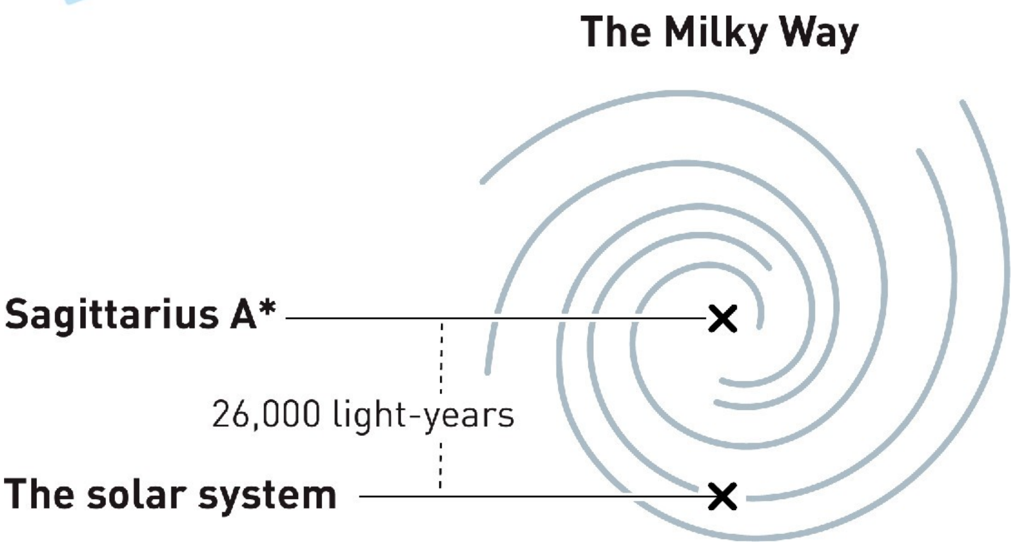


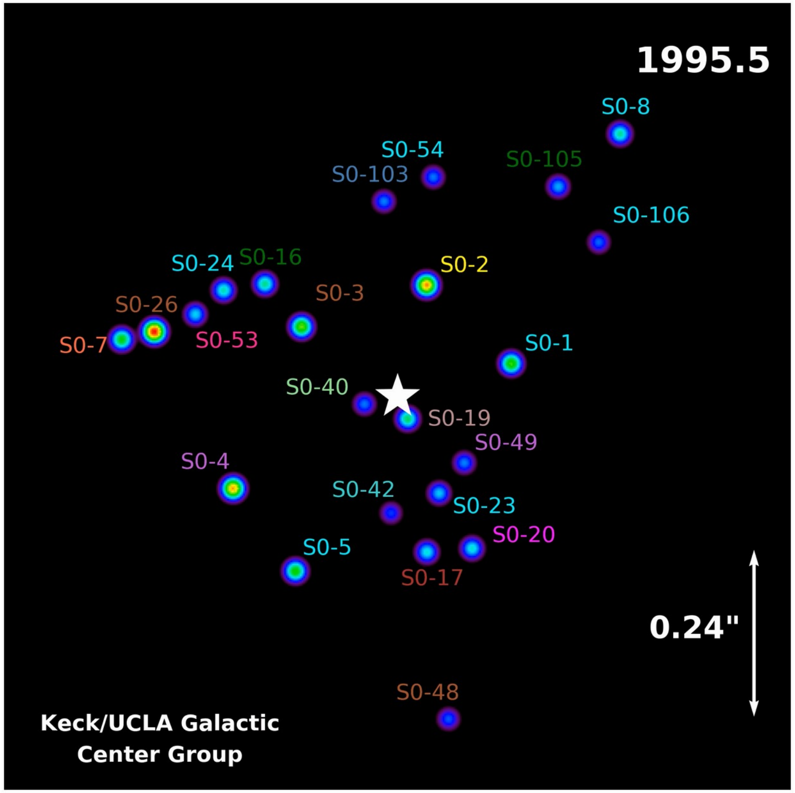
- Yukawa type correction with characteristic length scale λ_g:

$$\varphi(r) = \frac{GM}{r}\left(1 - e^{-\frac{r}{\lambda_g}}\right)$$

- LIGO bound from GW150914: $\boxed{\lambda_g > 1.6 \times 10^{13}\ \text{km};\ m_g \leq 1.2 \times 10^{-22}\ \text{eV}/c^2}$
- Solar System bounds: $\lambda_g > 1.6 \times 10^{10}$ km
- Binary-pulsar bounds: $\lambda_g > 2.8 \times 10^{12}$ km
- Expected detection limit for a future pulsar timing array with 300 pulsars, observed for 10 years (Lee et al. 2010, ApJ, 722, 1589): $m_g = 5 \times 10^{-23}$ eV

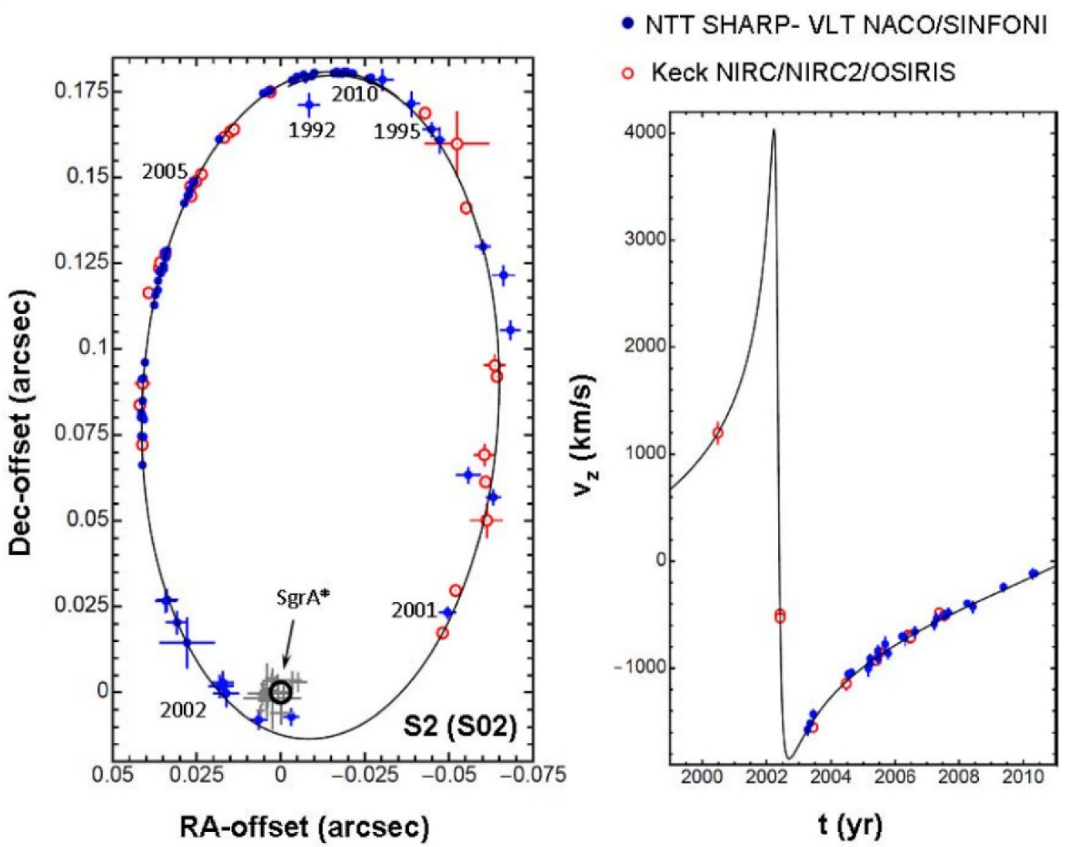
Stellar orbits around Sgr A*





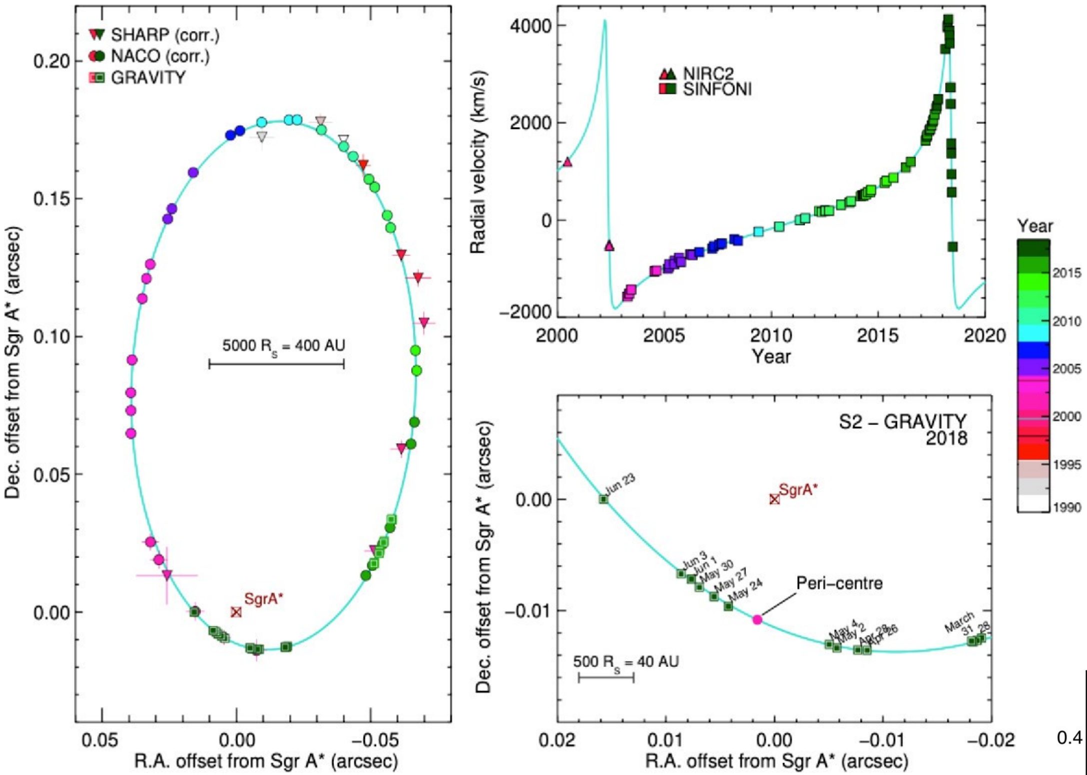
Stellar orbits monitored by 2 groups:

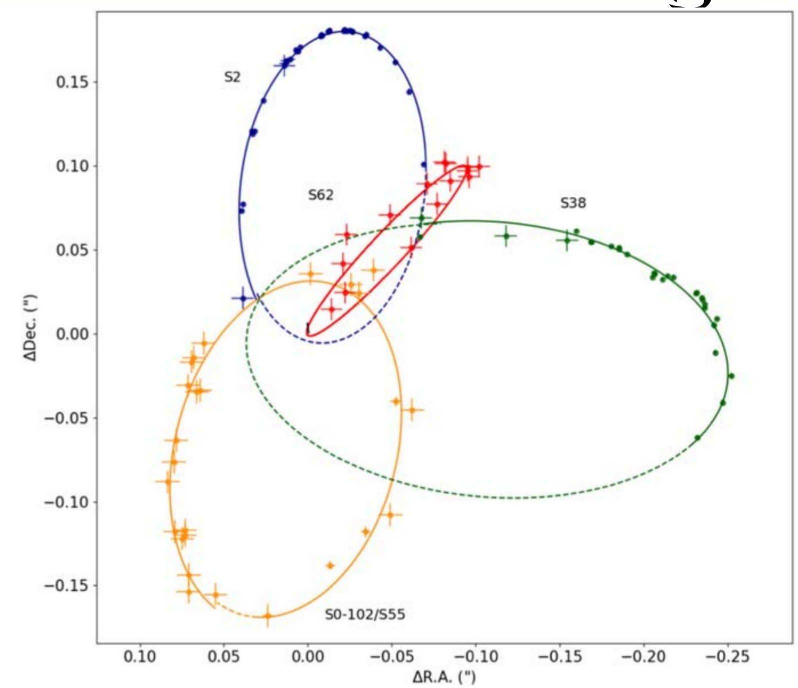
- R. Genzel (ESO): **New Technology Telescope** & **Very Large Telescope**, Chile
- A. Ghez: **Keck telescope**, Hawaii, USA

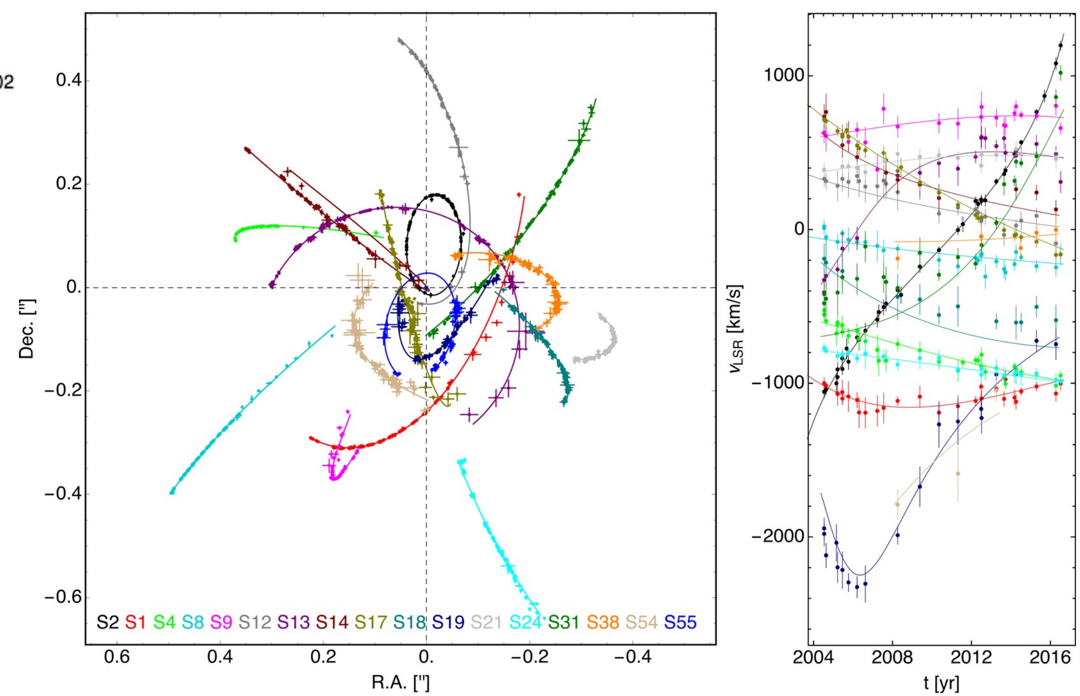


Observations of S2 star by NTT/VLT and Keck (Gillessen et al. 2009, ApJL, 707, L114; Genzel et al. 2010, Rev. Mod. Phys., 82, 3121)

New observations of stellar orbits around Sgr A*







- *Top left:* GRAVITY Collaboration, 2018, A&A, 615, L15
- *Top right:* Peißker et al. 2020, ApJ, 889:61
- *Bottom right:* Gillessen et al. 2017, ApJ, 837:30

Overiview of our results

- D. Borka, P. Jovanović, V. Borka Jovanović, A. F. Zakharov, *Constraining the range of Yukawa gravity interaction from S2 star orbits*, Journal of Cosmology and Astroparticle Physics, Vol. 2013, No. 11 (2013), p. 050.
- S. Capozziello, D. Borka, P. Jovanović, V. Borka Jovanović, *Constraining extended gravity models by S2 star orbits around the Galactic Centre*, Physical Review D, 90 (2014), p. 044052.
- A. F. Zakharov, P. Jovanović, D. Borka, V. Borka Jovanović, *Constraining the range of Yukawa gravity interaction from S2 star orbits II: bounds on graviton mass*, Journal of Cosmology and Astroparticle Physics, Vol. 2016, No. 05 (2016), p. 045.
- A. F. Zakharov, P. Jovanović, D. Borka, V. Borka Jovanović, *Constraining the range of Yukawa gravity interaction from S2 star orbits III: improvement expectations for graviton mass bounds*, Journal of Cosmology and Astroparticle Physics, Vol. 2018, No. 04 (2018), p. 050.
- P. Jovanović, D. Borka, V. Borka Jovanović, A. F. Zakharov, *Influence of bulk mass distribution on orbital precession of S2 star in Yukawa gravity*, European Physical Journal D 75:145, (2021), pp. 1-7.

Simulated orbits of S2 star in Yukawa gravity

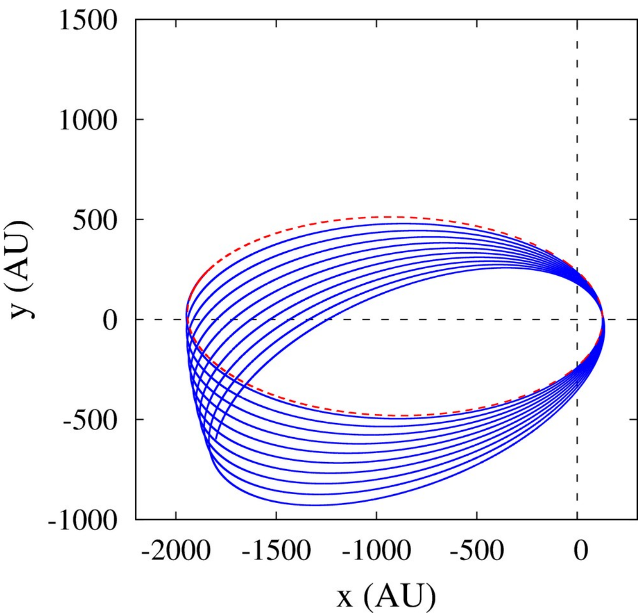
- Simulated orbits of S2 star in Yukawa gravitational potential obtained by numerical integration of differential equations of motion (Borka, Jovanović, Borka Jovanović, Zakharov, 2013, JCAP, 2013, No. 11, 050):

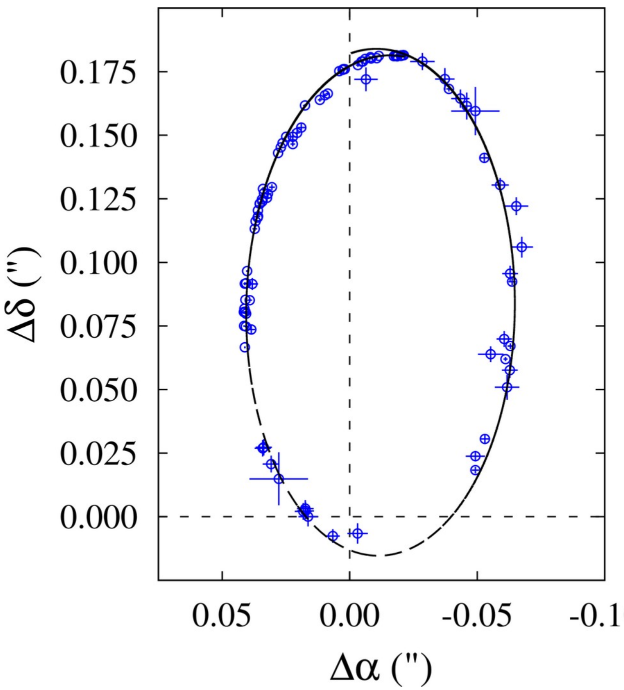
$$\dot{\mathbf{r}} = \mathbf{v}, \quad \mu\ddot{\mathbf{r}} = -\nabla\Phi(\mathbf{r})$$

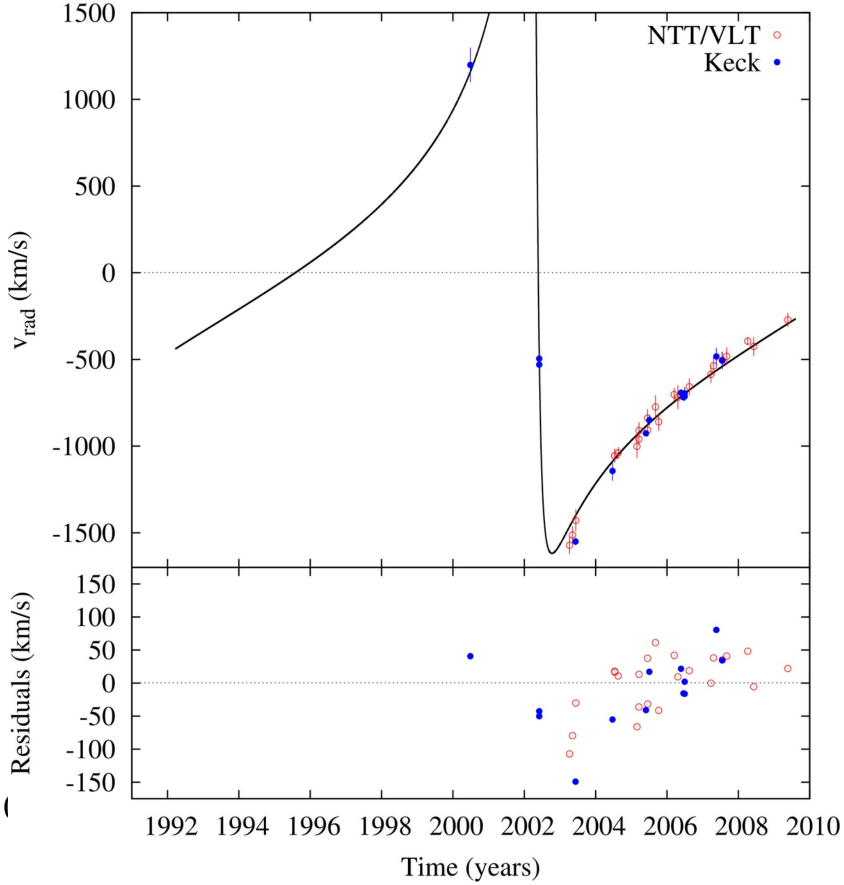
$$\Phi(r) = -\frac{GM}{(1+\delta)r}\left(1 + \delta e^{-\frac{r}{\Lambda}}\right)$$

- Simulated orbits were then fitted to the astrometric observations of S2 star

- Example for $\delta = 1/3$ (since it was successfully applied to clusters of galaxies)

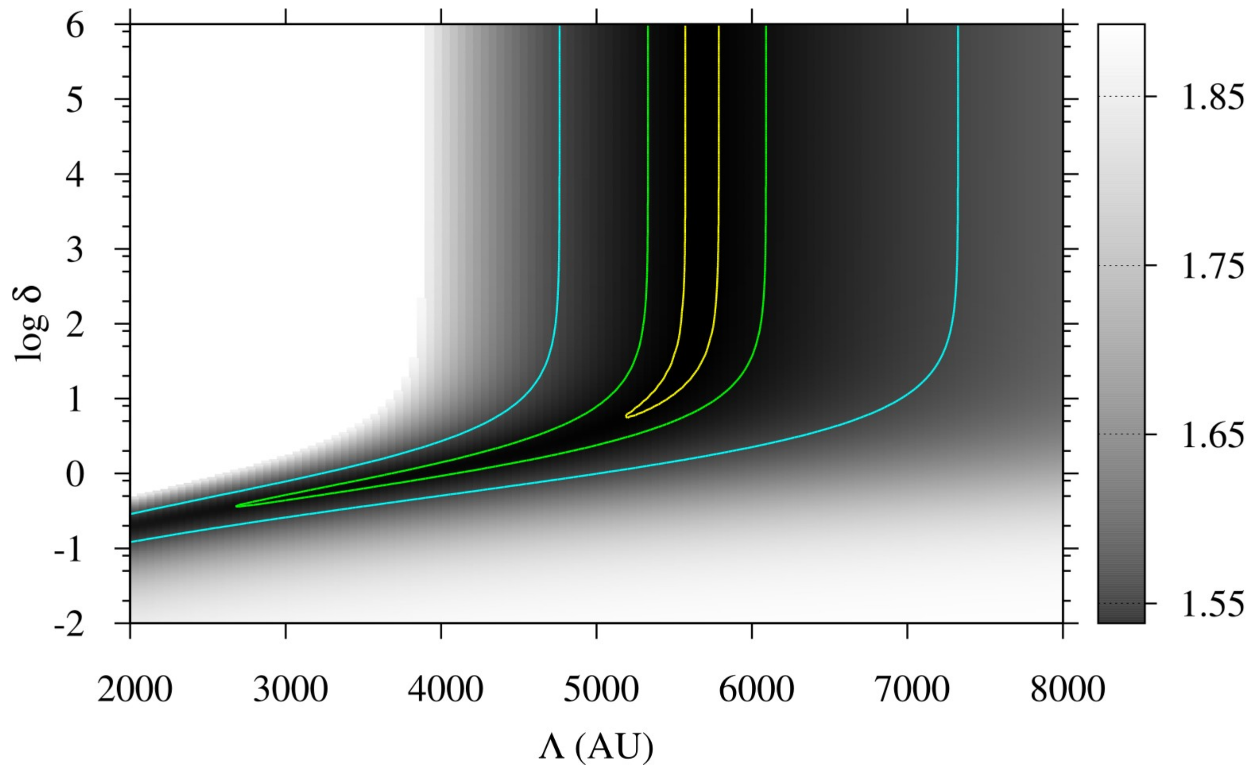






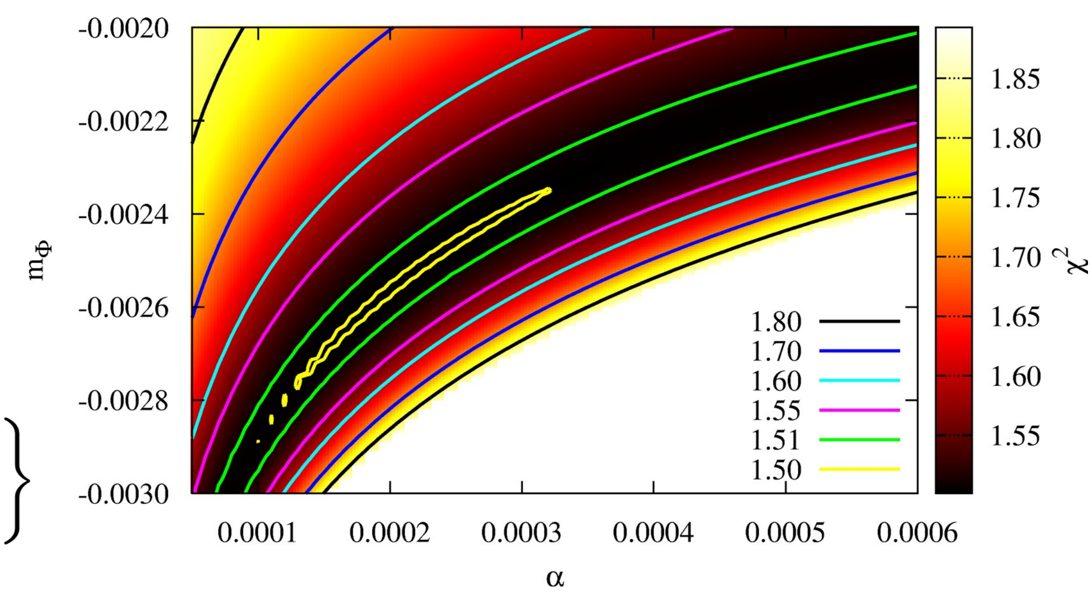
Reduced χ^2 maps of fitted S2 star orbits

- Reduced χ^2 map for Yukawa potential (Borka, Jovanović, Borka Jovanović, Zakharov, 2013, JCAP, 2013, No. 11, 050)

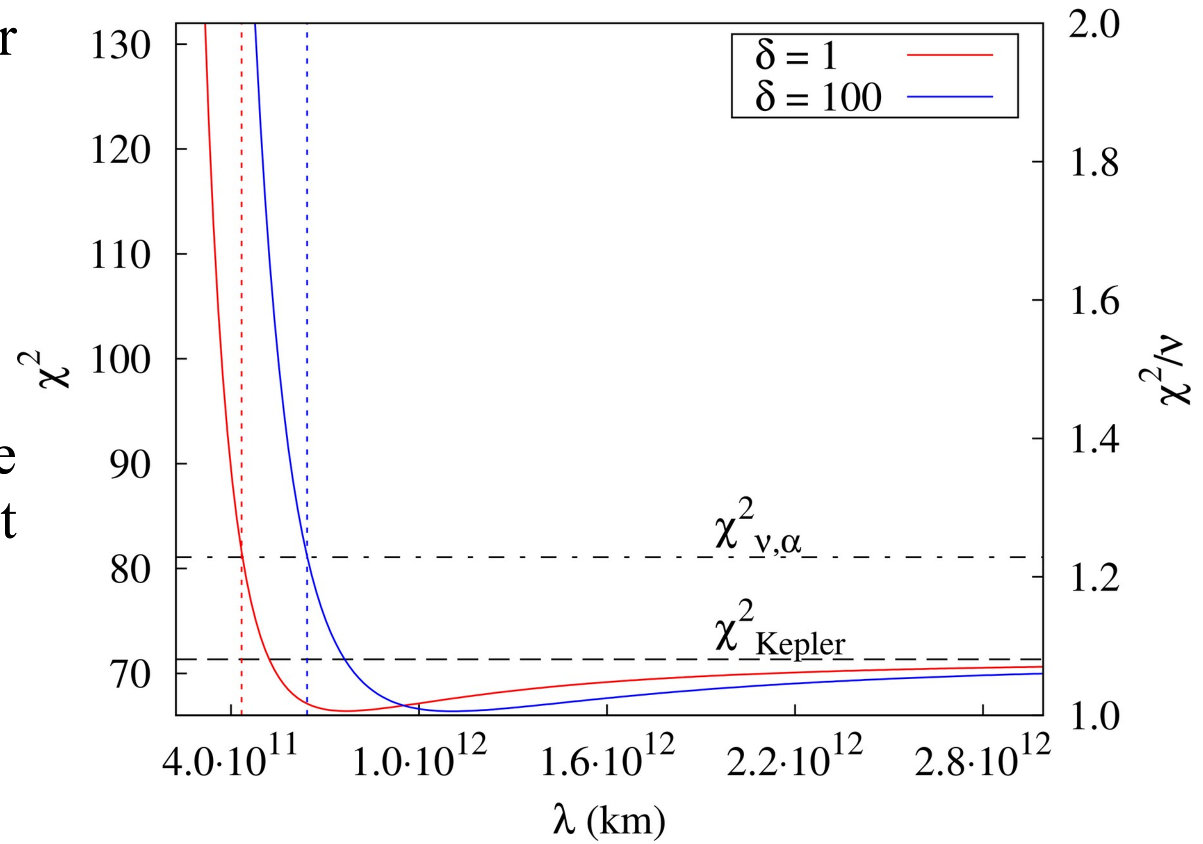


- Reduced χ^2 map for Sanders-like potential (Capozziello, Borka, Jovanović, Borka Jovanović, 2014, PRD, 90, 044052):

$$\Phi(\mathbf{x}) \; = \; -\frac{G_\infty M}{|\mathbf{x}|}\left\{1 + \alpha\, e^{-\sqrt{1-3\alpha}\, m_\phi |\mathbf{x}|}\right\}$$



Our estimates for graviton mass upper bound



- χ^2 test of goodness of the S2 star orbit fits by Yukawa potential
- Test statistic:

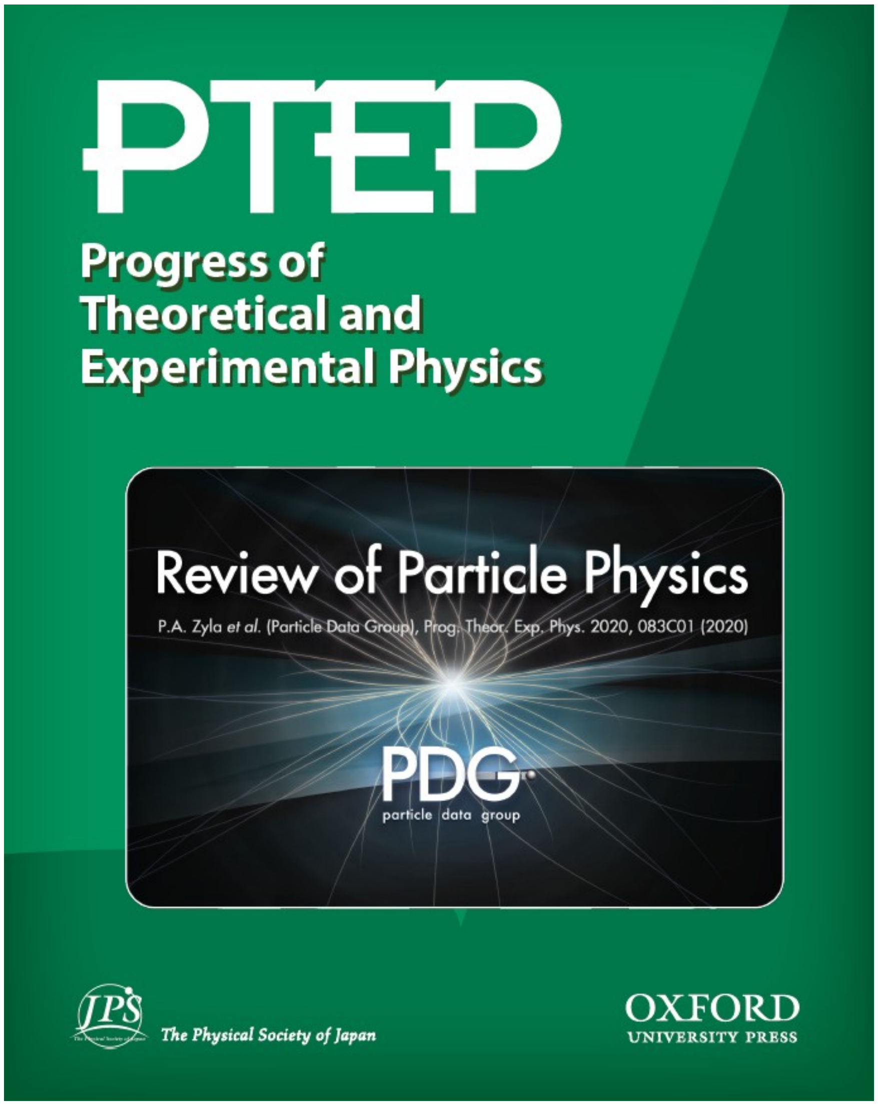
$$\chi^2 = \sum_{i=1}^{n} \left[\frac{(x_i^o - x_i^c)^2}{\sigma_{xi}^2 + \sigma_{int}^2} + \frac{(y_i^o - y_i^c)^2}{\sigma_{yi}^2 + \sigma_{int}^2} \right]$$

- Intrinsic dispersion of the data due to their mutually inconsistent uncertainties: $\sigma_{int} = 1.13$ mas
- NDOF: $\nu = 66$
- Significance level: $\alpha = 0.1$
- Critical value for χ^2: $\chi^2_{\nu,\alpha} = 81.08$
- Regions $\lambda < \lambda_x$ where $\chi^2 > \chi^2_{\nu,\alpha}$ can be excluded with 1 - α = 90% probability
- For $\delta = 1$: $\lambda_x = 2900 \pm 50\,\mathrm{AU} \approx 4.3 \times 10^{11}$ km
- For $\delta = 100$: $\lambda_x = 4300 \pm 50\,\mathrm{AU} \approx 6.4 \times 10^{11}$ km
- Corresponding upper bounds for graviton mass (Zakharov, Jovanović, Borka, Borka Jovanović, 2016, JCAP, 2016, No. 05, 045):

$$m_g = h\,c/\lambda_x \;\Rightarrow\; \boxed{m_g = 2.9 \times 10^{-21}\ \mathrm{eV} \quad \wedge \quad m_g = 1.9 \times 10^{-21}\ \mathrm{eV}}$$

Our estimates for graviton mass accepted by PDG

- From 2019, our estimate is in *Gauge and Higgs Boson Particle Listings* by PDG (Zyla et al., Particle Data Group, 2020, PTEP, 083C01)



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Gauge & Higgs Boson Particle Listings

γ, g, graviton, W

SIVARAM	95	AJP 63 473	C. Sivaram	(BANG)
FISCHBACH	94	PRL 73 514	E. Fischbach *et al.*	(PURD, JHU+)
RAFFELT	94	PR D50 7729	G. Raffelt	(MPIM)
CHERNIKOV	92	PRL 68 3383	M.A. Chernikov *et al.*	(ETH)
Also		PRL 69 2999 (erratum)	M.A. Chernikov *et al.*	(ETH)
COCCONI	92	AJP 60 750	G. Cocconi	(CERN)
COCCONI	88	PL B206 705	G. Cocconi	(CERN)
RYAN	85	PR D32 802	J.J. Ryan, F. Accetta, R.H. Austin	(PRIN)
CHIBISOV	76	SPU 19 624	G.V. Chibisov	(LEBD)
		Translated from UFN 119 551.		
DAVIS	75	PRL 35 1402	L. Davis, A.S. Goldhaber, M.M. Nieto	(CIT, STON+)
HOLLWEG	74	PRL 32 961	J.V. Hollweg	(NCAR)
FRANKEN	71	PRL 26 115	P.A. Franken, G.W. Ampulski	(MICH)
GOLDHABER	71B	RMP 43 277	A.S. Goldhaber, M.M. Nieto	(STON, BOHR, UCSB)
KROLL	71	PRL 26 1395	N.M. Kroll	(SLAC)
KROLL	71A	PRL 27 340	N.M. Kroll	(SLAC)
PARK	71	PRL 26 1393	D. Park, E.R. Williams	(WILC)
WILLIAMS	71	PRL 26 721	E.R. Williams, J.E. Faller, H.A. Hill	(WESL)
GOLDHABER	68	PRL 21 567	A.S. Goldhaber, M.M. Nieto	(STON)
YAMAGUCHI	59	PTPS 11 37	Y. Yamaguchi	

g or gluon

$I(J^P) = 0(1^-)$

SU(3) color octet

Mass $m = 0$. Theoretical value. A mass as large as a few MeV may not be precluded, see YNDURAIN 95.

VALUE	*DOCUMENT ID*		*TECN*	*COMMENT*
• • • We do not use the following data for averages, fits, limits, etc. • • •				
	ABREU	92E	DLPH	Spin 1, not 0
	ALEXANDER	91H	OPAL	Spin 1, not 0
	BEHREND	82D	CELL	Spin 1, not 0
	BERGER	80D	PLUT	Spin 1, not 0
	BRANDELIK	80C	TASS	Spin 1, not 0

gluon REFERENCES

YNDURAIN	95	PL B345 524	F.J. Yndurain	(MADU)
ABREU	92E	PL B274 498	P. Abreu *et al.*	(DELPHI Collab.)
ALEXANDER	91H	ZPHY C52 543	G. Alexander *et al.*	(OPAL Collab.)
BEHREND	82D	PL B110 329	H.J. Behrend *et al.*	(CELLO Collab.)
BERGER	80D	PL B97 459	C. Berger *et al.*	(PLUTO Collab.)
BRANDELIK	80C	PL B97 453	R. Brandelik *et al.*	(TASSO Collab.)

graviton

$J = 2$

graviton MASS

Van Dam and Veltman (VANDAM 70), Iwasaki (IWASAKI 70), and Zakharov (ZAKHAROV 70) almost simultanously showed that "... there is a discrete difference between the theory with zero-mass and a theory with finite mass, no matter how small as compared to all external momenta." The resolution of this "vDVZ discontinuity" has to do with whether the linear approximation is valid. De Rham *etal*. (DE-RHAM 11) have shown that nonlinear effects not captured in their linear treatment can give rise to a screening mechanism, allowing for massive gravity theories. See also GOLDHABER 10 and DE-RHAM 17 and references therein. Experimental limits have been set based on a Yukawa potential or signal dispersion. h_0 is the Hubble constant in units of 100 km s^{-1} Mpc^{-1}.

The following conversions are useful: $1\ \text{eV} = 1.783 \times 10^{-33}\ \text{g} = 1.957 \times 10^{-6}\ m_e$; $\lambda\!\!\!^{-}_C = (1.973 \times 10^{-7}\ \text{m}) \times (1\ \text{eV}/m_g)$.

VALUE (eV)	*DOCUMENT ID*		*TECN*	*COMMENT*
$<6 \times 10^{-32}$	[1] CHOUDHURY	04	YUKA	Weak gravitational lensing
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$<6.8 \times 10^{-23}$	BERNUS	19	YUKA	Planetary ephemeris INPOP17b
$<1.4 \times 10^{-29}$	[2] DESAI	18	YUKA	Gal cluster Abell 1689
$<5 \times 10^{-30}$	[3] GUPTA	18	YUKA	SPT-SZ
$<3 \times 10^{-30}$	[3] GUPTA	18	YUKA	Planck all-sky SZ
$<1.3 \times 10^{-29}$	[3] GUPTA	18	YUKA	redMaPPer SDSS-DR8
$<6 \times 10^{-30}$	[4] RANA	18	YUKA	Weak lensing in massive clusters
$<8 \times 10^{-30}$	[5] RANA	18	YUKA	SZ effect in massive clusters
$<7 \times 10^{-23}$	[6] ABBOTT	17	DISP	Combined dispersion limit from three BH mergers
$<1.2 \times 10^{-22}$	[6] ABBOTT	16	DISP	Combined dispersion limit from two BH mergers
$<2.9 \times 10^{-21}$	[7] ZAKHAROV	16	YUKA	S2 star orbit
$<5 \times 10^{-23}$	[8] BRITO	13		Spinning black holes bounds
$<4 \times 10^{-25}$	[9] BASKARAN	08		Graviton phase velocity fluctuations
$<6 \times 10^{-32}$	[10] GRUZINOV	05	YUKA	Solar System observations
$<9.0 \times 10^{-34}$	[11] GERSHTEIN	04		From Ω_{tot} value assuming RTG
$>6 \times 10^{-34}$	[12] DVALI	03		Horizon scales
$<8 \times 10^{-20}$	[13,14] FINN	02	DISP	Binary pulsar orbital period decrease
	[14,15] DAMOUR	91		Binary pulsar PSR 1913+16
$<7 \times 10^{-23}$	TALMADGE	88	YUKA	Solar system planetary astrometric data
$< 2 \times 10^{-29}\ h_0^{-1}$	GOLDHABER	74		Rich clusters
$<7 \times 10^{-28}$	HARE	73		Galaxy
$<8 \times 10^{4}$	HARE	73		2γ decay

[1] CHOUDHURY 04 concludes from a study of weak-lensing data that masses heavier than about the inverse of 100 Mpc seem to be ruled out if the gravitation field has the Yukawa form.

[2] DESAI 18 limit based on dynamical mass models of galaxy cluster Abell 1689.

[3] GUPTA 18 obtains graviton mass limits using stacked clusters from 3 disparate surveys.

[4] RANA 18 limit, 68% CL, obtained using weak lensing mass profiles out to the radius at which the cluster density falls to 200 times the critical density of the Universe. Limit is based on the fractional change between Newtonian and Yukawa accelerations for the 50 most massive galaxy clusters in the Local Cluster Substructure Survey. Limits for other CL's and other density cuts are also given.

[5] RANA 18 limit, 68% CL, obtained using mass measurements via the SZ effect out to the radius at which the cluster density falls to 500 times the critical density of the Universe for 182 optically confirmed galaxy clusters in an Altacama Cosmology Telescope survey. Limits for other CL's and other density cuts are also given.

[6] ABBOTT 16 and ABBOTT 17 assumed a dispersion relation for gravitational waves modified relative to GR.

[7] ZAKHAROV 16 constrains range of Yukawa gravity interaction from S2 star orbit about black hole at Galactic center. The limit is $< 2.9 \times 10^{-21}$ eV for $\delta = 100$.

[8] BRITO 13 explore massive graviton (spin-2) fluctuations around rotating black holes.

[9] BASKARAN 08 consider fluctuations in pulsar timing due to photon interactions ("surfing") with background gravitational waves.

[10] GRUZINOV 05 uses the DGP model (DVALI 00) showing that non-perturbative effects restore continuity with Einstein's equations as the gravition mass approaches 0, then bases his limit on Solar System observations.

[11] GERSHTEIN 04 use non-Einstein field relativistic theory of gravity (RTG), with a massive graviton, to obtain the 95% CL mass limit implied by the value of $\Omega_{tot} = 1.02 \pm 0.02$ current at the time of publication.

[12] DVALI 03 suggest scale of horizon distance via DGP model (DVALI 00). For a horizon distance of 3×10^{26} m (about age of Universe/c; GOLDHABER 10) this graviton mass limit is implied.

[13] FINN 02 analyze the orbital decay rates of PSR B1913+16 and PSR B1534+12 with a possible graviton mass as a parameter. The combined frequentist mass limit is at 90%CL.

[14] As of 2014, limits on dP/dt are now about 0.1% (see T. Damour, "Experimental tests of gravitational theory," in this *Review*).

[15] DAMOUR 91 is an analysis of the orbital period change in binary pulsar PSR 1913+16, and confirms the general relativity prediction to 0.8%. "The theoretical importance of the [rate of orbital period decay] measurement has long been recognized as a direct confirmation that the gravitational interaction propagates with velocity c (which is the immediate cause of the appearance of a damping force in the binary pulsar system) and thereby as a test of the existence of gravitational radiation and of its quadrupolar nature." TAYLOR 93 adds that orbital parameter studies now agree with general relativity to 0.5%, and set limits on the level of scalar contribution in the context of a family of tensor [spin 2]-biscalar theories.

graviton REFERENCES

BERNUS	19	PRL 123 161103	L. Bernus *et al.*	
DESAI	18	PL B778 325	S. Desai	(HYDER)
GUPTA	18	ANP 399 85	S. Gupta, S. Desai	
RANA	18	PL B781 220	A. Rana *et al.*	(DELHI)
ABBOTT	17	PRL 118 221101	B.P. Abbot *et al.*	(LIGO and Virgo Collabs.)
DE-RHAM	17	RMP 89 025004	C. de Rham *et al.*	
ABBOTT	16	PRL 116 061102	B.P. Abbott *et al.*	(LIGO and Virgo Collabs.)
ZAKHAROV	16	JCAP 1605 045	A.F. Zakharov *et al.*	
BRITO	13	PR D88 023514	R. Brito, V. Cardoso, P. Pani	(LISB, MISS, HSCA+)
DE-RHAM	11	PRL 106 231101	C. de Rham, G. Gabadadze, A.J. Tolley	
GOLDHABER	10	RMP 82 939	A.S. Goldhaber, M.M. Nieto	(STON, LANL)
BASKARAN	08	PR D78 044018	D. Baskaran *et al.*	
GRUZINOV	05	NAST 10 311	A. Gruzinov	(NYU)
CHOUDHURY	04	ASP 21 559	S.R. Choudhury *et al.*	(DELPH, MELB)
GERSHTEIN	04	PAN 67 1596	S.S. Gershtein *et al.*	(SERP)
		Translated from YAF 67 1618.		
DVALI	03	PR D68 024012	G.R. Dvali, A. Grizinov, M. Zaldarriaga	(NYU)
FINN	02	PR D65 044022	L.S. Finn, P.J. Sutton	
DVALI	00	PL B485 208	G.R. Dvali, G. Gabadadze, M. Porrati	(NYU)
TAYLOR	93	NAT 355 132	J.N. Taylor *et al.*	(PRIN, ARCBO, BURE+) J
DAMOUR	91	APJ 366 501	T. Damour, J.H. Taylor	(BURE, MEUD, PRIN)
TALMADGE	88	PRL 61 1159	C. Talmadge *et al.*	(JPL)
GOLDHABER	74	PR D9 1119	A.S. Goldhaber, M.M. Nieto	(LANL, STON)
HARE	73	CJP 51 431	M.G. Hare	(SASK)
IWASAKI	70	PR D2 2255	Y. Iwasaki	
VANDAM	70	NP B22 397	H. van Dam, M. Veltman	(UTRE)
ZAKHAROV	70	JETPL 12 312	V.I. Zakharov *et al.*	

W

$J = 1$

See the related review(s):
Mass and Width of the W Boson

W MASS

The W-mass listed here corresponds to the mass parameter in a Breit-Wigner distribution with mass-dependent width. To obtain the world average, common systematic uncertainties between experiments are properly taken into account. The LEP-2 average W mass based on published results is 80.376 ± 0.033 GeV [SCHAEL 13A]. The combined Tevatron data yields an average W mass of 80.387 ± 0.016 GeV [AALTONEN 13N]. A combination of the LEP average with this Tevatron average and the ATLAS value [AABOUD 18J], assuming a common systematic error of 7 MeV between the latter two [Jens Erler, 52nd Rencontres de Moriond

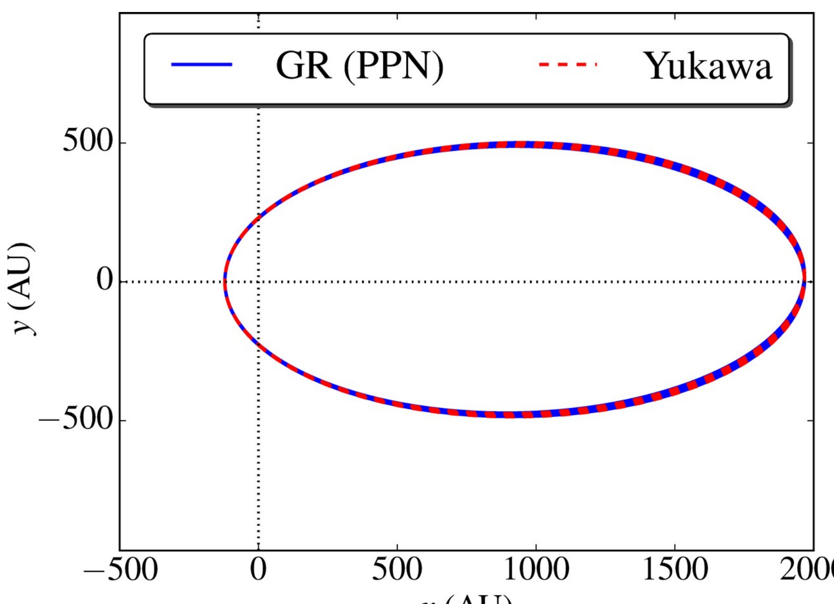
Possible improvements by future observations I

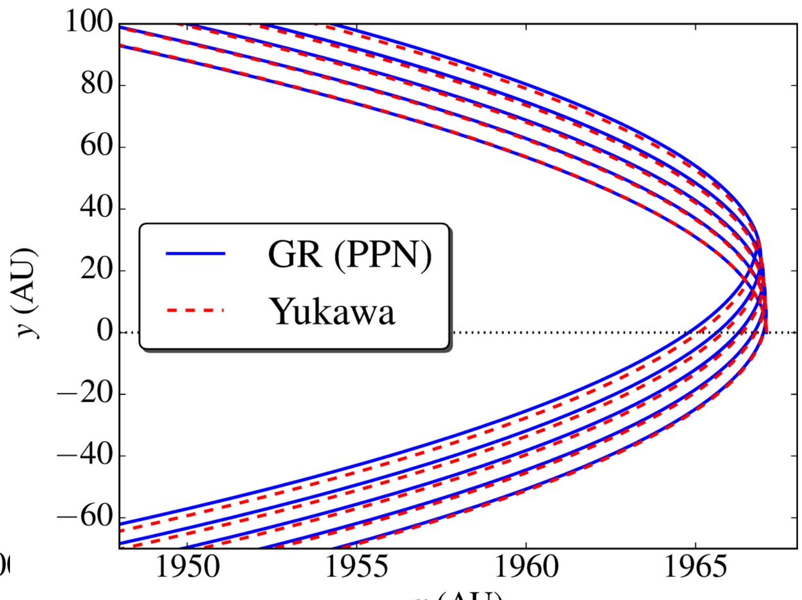
- Improvement expectations for graviton mass bounds, assuming that the GR predictions for orbital precession will be confirmed by the future observations
- Angle of orbital precession in Yukawa gravity: $\Delta\varphi_Y^{rad} \approx \frac{\pi\delta\sqrt{1-e^2}}{1+\delta}\frac{a^2}{\Lambda^2}, \quad a \ll \Lambda$
- Schwarzschild precession: $\Delta\varphi_{GR}^{rad} \approx \frac{6\pi GM}{c^2 a(1-e^2)}$

$$\Delta\varphi_Y = \Delta\varphi_{GR} \overset{\delta=1}{\Rightarrow} \boxed{\Lambda \approx \frac{c}{2}\sqrt{\frac{(a\sqrt{1-e^2})^3}{3GM}} \approx \sqrt{\frac{(a\sqrt{1-e^2})^3}{6R_S}} \approx \frac{T}{T_0}\sqrt{\frac{(a_0\sqrt{1-e^2})^3}{6R_S}},}$$

- Relative errors: $\frac{\Delta\Lambda}{\Lambda} = \frac{\Delta m_g}{m_g} \approx \pm\frac{3}{2}\left(\frac{|\Delta a|}{a} + \frac{e\,|\Delta e|}{1-e^2} + \frac{1}{3}\frac{|\Delta M|}{M}\right)$

where T_0 and a_0 correspond to a selected orbit (e.g. of S2 star)





(Zakharov, Jovanović, Borka, Borka Jovanović, 2018, JCAP, 2018, No. 04, 050)

Possible improvements by future observations II

Star name	T_{Kep} (yr)	$\Delta\varphi$ ($''$)	Δs (mas)	$\Lambda \pm \Delta\Lambda$ (AU)	$m_g \pm \Delta m_g$ (10^{-24} eV)	R.E. (%)
S1	168.4	48.2	0.22	369952.9 ± 43820.4	22.4 ± 2.7	11.8
S2	16.3	722.1	0.83	15125.5 ± 884.7	547.9 ± 32.0	5.8
S4	78.2	65.5	0.16	200418.3 ± 11191.1	41.4 ± 2.3	5.6
S6	195.5	102.4	0.60	226607.6 ± 8807.8	36.6 ± 1.4	3.9
S8	94.4	138.0	0.49	125957.9 ± 8420.6	65.8 ± 4.4	6.7
S9	52.2	124.3	0.27	101193.1 ± 9289.8	81.9 ± 7.5	9.2
S12	59.9	314.6	0.86	54047.1 ± 3026.3	153.3 ± 8.6	5.6
S13	49.8	91.6	0.17	124334.4 ± 5855.1	66.7 ± 3.1	4.7
S14	56.2	1465.9	4.02	16508.7 ± 2802.9	502.0 ± 85.2	17.0
S17	77.9	66.1	0.16	198588.4 ± 16771.2	41.7 ± 3.5	8.4
S18	42.6	107.1	0.18	102263.2 ± 5784.8	81.0 ± 4.6	5.7
S19	137.6	87.1	0.38	214568.1 ± 89676.6	38.6 ± 16.1	41.8
S21	37.6	217.4	0.41	56500.3 ± 4881.5	146.7 ± 12.7	8.6
S22	550.0	19.0	0.17	1347095.9 ± 580678.1	6.2 ± 2.7	43.1
S23	46.7	114.1	0.22	102079.9 ± 28448.6	81.2 ± 22.6	27.9
S24	336.5	107.5	0.93	286723.3 ± 41927.1	28.9 ± 4.2	14.6
S29	102.7	98.5	0.35	169074.0 ± 37807.8	49.0 ± 11.0	22.4
S31	110.4	63.3	0.21	244347.7 ± 17733.9	33.9 ± 2.5	7.3
S33	195.3	47.9	0.25	400731.7 ± 75407.3	20.7 ± 3.9	18.8
S38	19.5	427.5	0.53	24533.9 ± 1005.0	337.8 ± 13.8	4.1
S39	82.6	364.5	1.26	56824.5 ± 6638.4	145.8 ± 17.0	11.7
S42	339.7	30.8	0.22	736342.1 ± 312551.0	11.3 ± 4.8	42.4
S54	482.2	81.5	0.90	422181.5 ± 692183.8	19.6 ± 32.2	164.0
S55	13.0	382.8	0.34	21721.6 ± 1465.5	381.5 ± 25.7	6.7
S60	88.6	105.5	0.34	149149.2 ± 11131.0	55.6 ± 4.1	7.5
S66	675.3	13.4	0.11	1933974.1 ± 269754.5	4.3 ± 0.6	13.9
S67	438.3	19.3	0.14	1188222.1 ± 116748.7	7.0 ± 0.7	9.8
S71	352.1	106.2	0.95	295890.2 ± 56006.2	28.0 ± 5.3	18.9
S83	667.2	15.3	0.15	1737943.9 ± 477698.2	4.8 ± 1.3	27.5
S85	3619.1	11.0	0.44	5195117.0 ± 8106789.5	1.6 ± 2.5	156.0
S87	1663.8	7.6	0.12	4641782.4 ± 619016.2	1.8 ± 0.2	13.3
S89	412.3	31.0	0.27	806547.1 ± 140413.3	10.3 ± 1.8	17.4
S91	973.6	11.4	0.14	2626592.1 ± 322729.8	3.2 ± 0.4	12.3
S96	673.2	13.6	0.12	1907722.2 ± 189196.9	4.3 ± 0.4	9.9
S97	1296.3	9.7	0.15	3407955.4 ± 1361276.1	2.4 ± 1.0	39.9
S145	434.8	23.6	0.19	1016130.7 ± 535791.9	8.2 ± 4.3	52.7
S175	97.7	1812.0	7.23	18570.1 ± 5168.9	446.3 ± 124.2	27.8
R34	893.3	18.6	0.27	1741793.7 ± 558192.6	4.8 ± 1.5	32.0
R44	2825.3	5.5	0.13	7740489.4 ± 5361256.0	1.1 ± 0.7	69.3

(Zakharov, Jovanović, Borka, Borka Jovanović, 2018, JCAP, 2018, No. 04, 050)

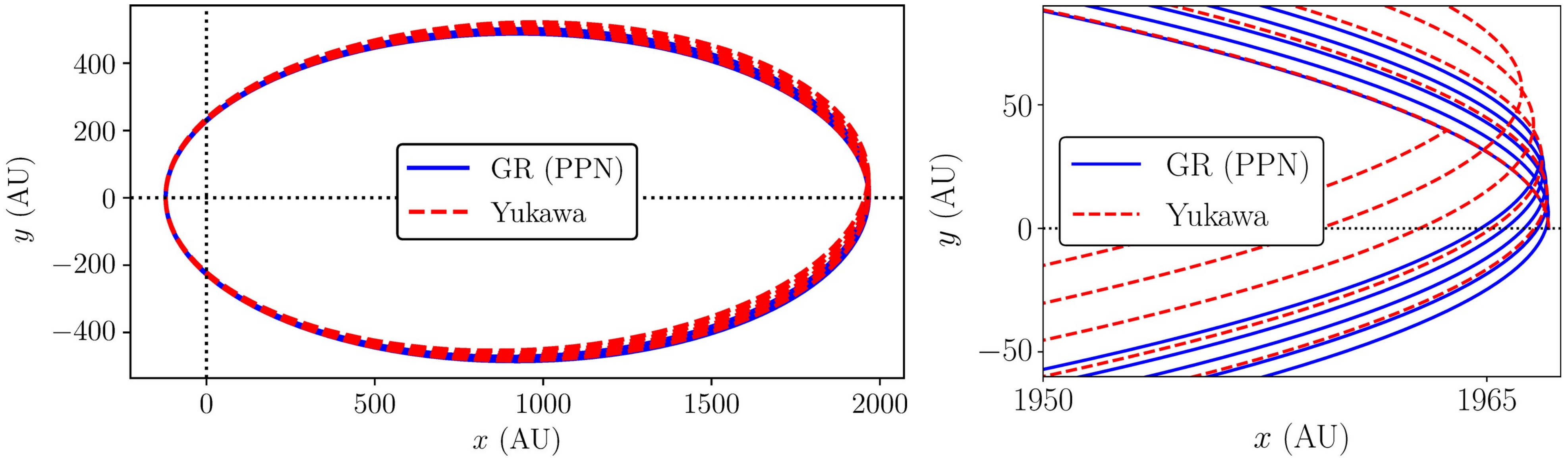
- Orbits with small eccentricities provide better constraints on graviton mass
- Due to linear dependence of Λ on orbital period, future monitorings of a bright star with a period around ≈ 50 years and small eccentricity could provide graviton mass constraint of: $m_g < 5 \times 10^{-23}$ eV
- In the case of S2 star, this estimate could reach: $m_g \approx 5.48 \times 10^{-22}$ eV

Influence of bulk distribution of matter I

- Effects of bulk mass distribution on orbital precession of S2 star in Yukawa gravity (Jovanović, Borka, Borka Jovanović, Zakharov, 2021, EPJD, 75, 145)
- Bulk distribution of matter which decribes stellar cluster, interstellar gas and dark matter contained within some radius *r* around SMBH: $M(r) = M_{BH} + M_{ext}(r)$
- Double power-law mass density profile (Genzel et al. 2003, ApJ, 594, 812):

$$\rho(r) = \rho_0 \left(\frac{r}{r_0}\right)^{-\alpha}, \ \alpha = \begin{cases} 2.0 \pm 0.1, & r \geq r_0 \\ 1.4 \pm 0.1, & r < r_0 \end{cases} \Rightarrow M_{ext}(r) = \frac{4\pi\rho_0 r_0^\alpha}{3-\alpha} r^{3-\alpha}$$

- For S2 star: $r_0 = 10'' \ \wedge \ \alpha = 1.4$
- Orbital precession of S2 star in Yukawa gravity for different mass densities of matter



- Example for: $\lambda = 3130$ AU, $\delta = 1 \ \wedge \ \rho_0 = 2 \times 10^8 \ M_\odot \ \mathrm{pc}^{-3}$

Influence of bulk distribution of matter II

- The case when S2 star orbital precession in Yukawa gravity with extended mass density distribution is the same as in GR ($0^{\circ}.18$)

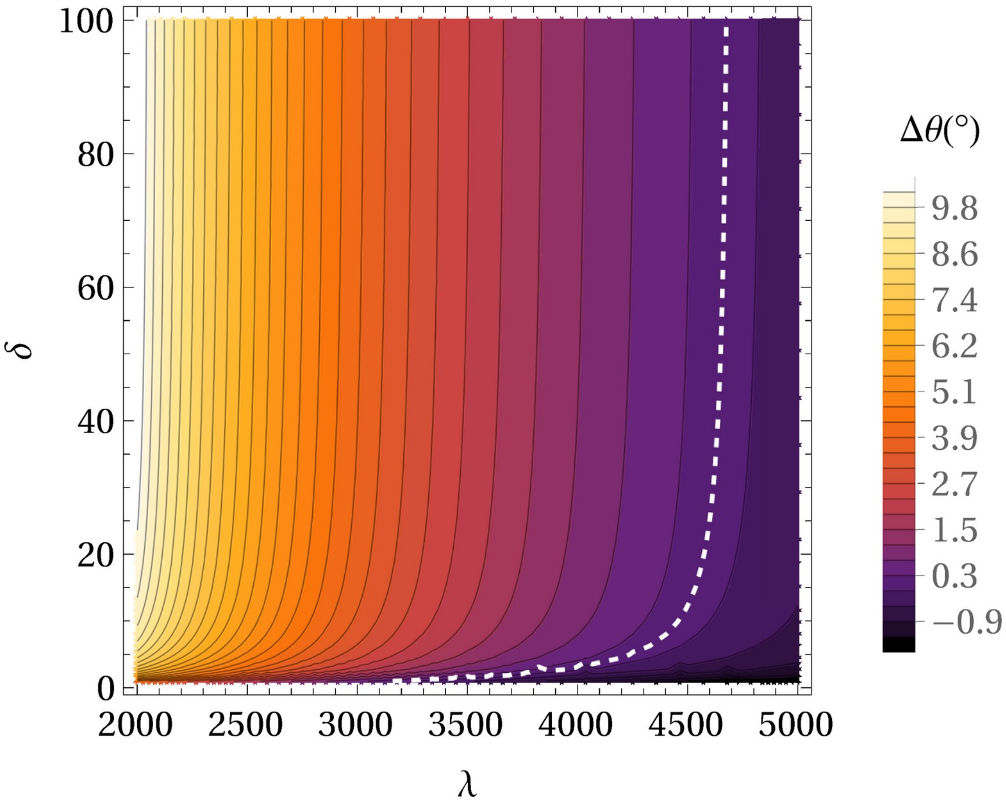


Table 1 The values of parameter λ (in AU) for different combinations of 3 values of parameter δ the 5 values of the mass density distribution of extended matter ρ_0

	ρ_0 (in $10^8 M_\odot \mathrm{pc}^{-3}$)				
	0	2	4	6	8
δ=1	15125	3130	2080	1597	1302
δ=10	20395	4425	3015	2370	1978
δ=100	21285	4640	3175	2500	2090

Table 2 The graviton mass (m_g) estimates corresponding to all mass density distributions presented in Table 1, in the case when Yukawa gravity parameter $\delta = 1$

ρ_0 (in $10^8 M_\odot \mathrm{pc}^{-3}$)	0	2	4	6	8
m_g (in 10^{-21} eV)	0.5	2.6	4.0	5.2	6.4

- S2 star recession per orbital period in parameter space of Yukawa gravity for mass density distribution of extended matter: $\rho_0 = 2 \times 10^8 \ M_\odot \mathrm{pc}^{-3}$

(Jovanović, Borka, Borka Jovanović, Zakharov, 2021, EPJD, 75, 145)

- For larger mass density distributions of the extended matter, the corresponding estimates for graviton mass could be slightly larger but still in the expected interval

Conclusions

- Analysis of the observed stellar orbits around Sgr A* in the frame of Yukawa gravity represents a powerful tool for constraining the graviton mass and testing the GR predictions.
- Fitting the simulated stellar orbits in Yukawa potential to the observed orbit of S2 star showed that the range of Yukawa interaction Λ is on the order of several thousand astronomical units (AU)
- Assuming that Λ corresponds to the Compton wavelength of graviton λ_g, we estimated the upper bound for its mass to: $m_g \le 2.9 \times 10^{-21}$ eV
- Our estimate for graviton mass upper bound is consistent with the LIGO results, but obtained in an independent way, and since 2019. it is included in the *Gauge and Higgs Boson Particle Listings* published by PDG
- Range of Yukawa gravity Λ can be constrained in such a way to induce the same orbital precession of stellar orbits as in GR
- Monitoring of a bright S-star with orbital period of $\approx$ 50 years and small eccentricity by the future telescopes could provide an opportunity to constrain the upper bound for graviton mass to: $m_g < 5 \times 10^{-23}$ eV
- Estimates for graviton mass may be slightly larger for larger mass density distributions of the extended matter, but are still in the expected interval

Thank you for your attention!