Thermodynamics and Complexity of Holographic Backgrounds

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SEENET-MTP WORKSHOP BEYOND THE STANDARD MODEL(S)
BW2021

7 - 10 September 2021, Belgrade, Serbia

Outline

- Black Hole Thermodynamics
- Extended Thermodynamic Framework
- Thermodynamic Information Geometry
- Complexity of Black Holes
- Applications to 3d Holographic Models

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First Law of Black Hole Thermodynamics

- Thermodynamic potential: E or M.
- Extensive parameters: $E^a = (S, J, Q, \dots)$.
- Intensive parameters: $I^a = (T, \Omega, \Phi, \dots)$.
- The first law (balance of energy):

$$dM = TdS + \Omega dJ + \Phi dQ + \dots = \delta_{ab} I^a dE^b. \qquad (1)$$

- Condition for thermodynamic equilibrium:

$$\frac{\partial M}{\partial E^a} = \delta_{ab} I^b. \qquad (2)$$

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Asymptotically Flat Spacetimes

Killing symmetries:

$$\xi = \xi_{(t)} + \Omega \xi_{(\phi)}. \tag{3}$$

Komar integrals:

$$M = -\frac{D-2}{16\pi(D-3)} \int\limits_{S_\infty} *d\xi_{(t)}, \quad J = \frac{1}{16\pi} \int\limits_{S_\infty} *d\xi_{(\phi)}. \tag{4}$$

The Hawking temperature on the horizon:

$$T = \frac{\kappa}{2\pi} = \frac{1}{2\pi} \sqrt{-\frac{1}{2} (\nabla_\mu \xi_\nu)(\nabla^\mu \xi^\nu)}\Big|_H. \tag{5}$$

Rigidity theorem on the horizon:

$$\xi_\mu \xi^\mu\Big|_H = 0 \quad \Rightarrow \quad \Omega = const. \tag{6}$$

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Wald's Entropy

Wald '93:

$$S = -2\pi \int_H \sqrt{|h|} \frac{\delta \mathcal{L}}{\delta R_{\mu\nu\rho\sigma}} \epsilon_{\mu\nu} \epsilon_{\rho\sigma} d^{d-1}x. \tag{7}$$

Includes higher derivative theories:

$$\frac{\delta \mathcal{L}}{\delta R_{\mu\nu\rho\sigma}} = \frac{\partial \mathcal{L}}{\partial R_{\mu\nu\rho\sigma}} - \nabla_{\zeta_1} \left(\frac{\partial \mathcal{L}}{\partial \nabla_{\zeta^1} R_{\mu\nu\rho\sigma}} \right) + \cdots$$
$$+ (-1)^n \nabla_{\zeta_1} \ldots \nabla_{\zeta_n} \left(\frac{\partial \mathcal{L}}{\partial \nabla_{(\zeta_1} \ldots \nabla_{\zeta_n)} R_{\mu\nu\rho\sigma}} \right). \tag{8}$$

The antisymmetric normal bivector to the surface H:

$$\nabla_\mu \xi_\nu = \kappa \epsilon_{\mu\nu}, \quad \epsilon_{\mu\nu} = -\epsilon_{\nu\mu}, \quad \epsilon_{\mu\nu} \epsilon^{\mu\nu} = -2. \tag{9}$$

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Extended Thermodynamic Framework

Consider Λ as an effective parameter. One can show that it behaves like a pressure (Kastor, Ray & Traschen '09):

$$P = -\frac{D-2}{16\pi}\Lambda. \tag{10}$$

The first law is modified:

$$dM = TdS + \Omega dJ + \Phi dQ + VdP. \tag{11}$$

Thermodynamic volume:

$$V = -\int *\omega, \quad d*d\xi + 2\Lambda d*\omega, \quad \xi = *d*\omega. \tag{12}$$

Komar integrals:

$$M = -\frac{D-2}{16\pi(D-3)}\int_{S_\infty}(*d\xi_{(t)} + 2\Lambda * \omega), \quad J = \frac{1}{16\pi}\int_{S_\infty} *d\xi_{(\phi)}. \tag{13}$$

Thermodynamic Information Geometry

How to define a proper distance between equilibrium macrostates in the phase space $\mathcal{E} = \{\Xi, E^a\}$?

Fluctuation theory (Ruppeiner '79):

$$S(E^a) = S_0 + EQL + \frac{\partial^2 S}{\partial E^a \partial E^b} dE^a dE^b + \cdots$$

Ruppeiner information metric:

$$g_{ab}^{(R)} = -\frac{\partial^2 S}{\partial E^a \partial E^b} = -\mathrm{Hess} S(\vec{E}). \qquad (14)$$

Weinhold information metric (Weinhold '75):

$$g_{ab}^{(W)} = \frac{\partial^2 M}{\partial E^a \partial E^b} = \mathrm{Hess} M(\vec{E}). \qquad (15)$$

Legendre invariant metrics (H. Quevedo '17):

$$ds^2 = \left(\delta_{ac}\xi^{cb}E^a\frac{\partial\Xi}{\partial E^b}\right)\left(\eta_e^d\frac{\partial^2\Xi}{\partial E^d\partial E^f}dE^e dE^f\right) \tag{16}$$

New thermodynamic geometry (Mansoori & Mirza '19):

$$\hat{g} = \mathrm{blockdiag}\left(\frac{1}{T}\frac{\partial^2 M}{\partial S^2}, -\hat{G}\right), \tag{17}$$

$$G_{ij} = \frac{1}{T}\frac{\partial^2 M}{\partial Y^i\partial Y^j}, \quad Y^i = (Q_1, \ldots, Q_m). \tag{18}$$

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Complexity

Computational complexity is a measure of how hard is to implement a given task:

- The computational complexity of an algorithm is the amount of resources required to run it.
- The complexity of a quantum state is the minimal number of simple unitary operations required to prepare it from another reference state.
- To date there is no well-posed definition for the complexity for quantum field theory states.

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Complexity in Holography

- Several proposals for the dual of quantum complexity: complexity equals volume" (CV) and "complexity equals action" (CA) and others.
- Acording to CV the complexity of the CFT state is holographically dual to the growth of the size of the black hole.
- Acording to the CA the quantum complexity of ground state of CFT is given by the classical gravitational action evaluated on the "Wheeler-DeWitt patch" (WDW).
- The Lloyd bound on the growth rate of complexity:

$$\dot{C} \leq (F + TS)_+ - (F + TS)_+ . \tag{19}$$

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Thermodynamic Geometry and 3d Holographic Models

❶ Thermodynamic information geometry for 3d $WAdS_3$ black hole solution in TMG dual to $WCFT_2$ with left and right central charges → critical temperature (Dimov, Rashkov & Vetsov '19):

$$T_c = \frac{1}{\pi(c_L + \sqrt{c_L c_R})}. \tag{20}$$

❷ Thermodynamic information geometry for 3d $SLifBH_3$ black hole solution in NMG dual to an unknown CFT_2 → quantum covariance matrix (Kolev, Staykov & Vetsov '19):

$$\mathcal{G}_{ij} = \frac{\partial^2 \psi}{\partial \lambda^i \partial \lambda^j} = \langle (X_i - \langle X_i \rangle)\,(X_j - \langle X_j \rangle) \rangle\,. \tag{21}$$

Complexity and 3d Holographic Models

1. Complexity growth rate for 3d $WAdS_3$ black hole solution in TMG dual to $WCFT_2$ with left and right central charges $\rightarrow$ non-trivial bound on the mass (Dimov, Rashkov & Vetsov '19):
$$M \geq \frac{(2c_L - c_R)\sqrt{c_L}}{3\sqrt{12c_R - 15c_L}}. \tag{22}$$
2. Complexity growth rate for 3d rotating BTZ-EGB black hole solution with Gauss-Bonnet coupling α lead to the determinantion of the closest distance ϵ to the curvature singularity (Dimov, Iliev, Radomirov, Rashkov & Vetsov '21 upcoming):
$$\epsilon \geq f(\alpha). \tag{23}$$

Thank You!

- In collaboration with:

  Prof. DSc Radoslav Rashkov,
  Prof. Hristo Dimov,
  Dr. Miroslav Radomirov,
  Dr. Ivo Iliev.

- Supported by
  – The Bulgarian NSF Grant N28/5.

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